Paper Name: Circuit Theory & Network
Paper Code: EC302
Contacts: 4L
Credit: 4

Course Content

Subject Name: Circuit Theory & Networks
Subject Code: EC 302
Credit Value: 4.0
Length: 42.0 classes
Pre requisite: Basic Electrical Engineering, Basic Electronics Engineering, Laplace Transform
Contact Hours: (3L + 1T)

Course Objective:
Circuit Theory & Networks is an introductory course in electrical engineering and application of basics of electrical engineering in solving electrical networks. Students are introduced to simple applied electrical circuits with dc & ac sources, theories and practice to impart skill set to have visualization of electrical engineering applications. It is a course suitable for students pursuing electrical engineering as well as other related engineering disciplines.

Learning Outcomes:

Upon successful completion of this course, the students should have the basic understanding of the following topics:

1. Concept of RLC series & parallel circuits, behaviour and resonance. Application of resonance in electronic engineering.
2. Definition of various kinds of active sources and their symbols, Kirchhoff's Laws and Networks theorems for complex circuit analysis with both DC & AC sources.
3. Magnetically coupled circuits & their analysis.
4. Application of Laplace transform in solving electrical networks under transient & steady state conditions.
5. Basic concept of Graph theory & its application in solving electrical networks.
6. Two port network parameter calculation and application in electrical systems.

Lesson Plan:

Module No.	Total No of Lectures	No. of Lecture	Topics	Applications	Book(s) Follow	Topics for Self Study
Module-I Resonance Page (6- 12)	**5L**	**2L**	Series and Parallel resonance, Impedance & Admittance Characteristics	Brief idea about behaviour of RLC series & parallel circuit, resonance & its practical applications.	1. Circuit Theory & Network Analysis by A. Chakraborti (Dhanpat Rai Publications) 2. Network Analysis & Synthesis by Ravish R. Singh (Mcgraw hill Publications) 3. Circuits and Networks Analysis and Synthesis by Sudhakar	RLC series/ parallel circuits, behaviour, condition for resonance.
		3L	Properties of resonance, Quality Factor, Half Power Points, Bandwidth, Phasor diagrams, Transform diagrams, Practical resonant circuits, Solution of Problems			
Module-II Network Analysis Page (13– 36)	**12L**	**2L**	Kirchoff's Current law, Formulation of Node equations and solutions,Solution of problems with DC and AC sources	Analysis of Electric Circuits with DC & AC sources	1. Circuit Theory & Network Analysis by A. Chakraborti (Dhanpat Rai Publications) 2. Network Analysis & Synthesis by Ravish R. Singh (Mcgraw hill Publications) 3. Circuits and Networks Analysis and Synthesis by Sudhakar	Network theorems with DC sources
		2L	Mesh Current Analysis:Kirchoff's Voltage law, Formulation of mesh equations , Solution of mesh equations by Cramer's rule and matrix method , Solution of problems with DC and AC sources			
		8L	Network Theorems: Definition and Implication of Superposition Theorem , Thevenin's theorem, Norton's theorem ,			

			Reciprocity theorem, Compensation theorem , maximum Power Transfer theorem, Millman's theorem, Star delta transformations, Tellegen's Theorem, Solutions and problems with DC and AC sources, driving point admittance, transfer Admittance, Driving point impedance, Transfer impedance.			
Module-III Graph Theory Page (37 - 41)	**4L**	1L	Concept of Tree, Branch, Tree link	Application of Graph Theory in solving Electric Circuits	1. Circuit Theory & Network Analysis by A. Chakraborti (Dhanpat Rai Publications) 2. Network Analysis & Synthesis by Ravish R. Singh (Mcgraw hill Publications)	Concept of tree, branch, twig, link etc.
		3L	Incidence Matrix, Cut Set Matrix, Tie Set Matrix, Formation of incidence, tie set, cut set matrices of electric circuits			
Module-IV Magnetically Coupled Circuits Page (42 –45)	**4L**	2L	Magnetic coupling, Polarity of coils, Polarity of induced voltage, Concept of Self and Mutual inductance, Coefficient of coupling	Modelling & Analysis of coupled circuits	1. Circuit Theory & Network Analysis by A. Chakraborti (Dhanpat Rai Publications) 2. Network Analysis & Synthesis by Ravish R. Singh (Mcgraw hill	Self & mutual inductance, DOT convention.
		2L	Modelling of coupled circuits, Solution of problems.			

					Publications) 3. Circuits and Networks Analysis and Synthesis by Sudhakar 4. Electronic Circuit Analysis by Rao (Pearson Publication)	
Module-V Laplace Transform Page (46 - 50)	**7L**	2L	Definition Of Laplace Transform, Advantages, Initial Value theorem and final value theorem, Poles, zeros, transfer function	Properties of Laplace transform	1. Circuit Theory & Network Analysis by A. Chakraborti (Dhanpat Rai Publications) 2. Network Analysis & Synthesis by Ravish R. Singh (Mcgraw hill Publications) 3. Circuits and Networks Analysis and Synthesis by Sudhakar	Initial & Final value theorem, poles, zeros & transfer function.
		2L	Laplace Transform of different types of signals			
		3L	Inverse Laplace Transform using partial fraction method, circuit analysis in s-domain			
Module-VI Transient Analysis Page (51 - 59)	**5L**	3L	Transient analysis of RC, RL, RLC circuit with DC & AC sources		1. Circuit Theory & Network Analysis by A. Chakraborti (Dhanpat Rai Publications) 2. Network Analysis & Synthesis by Ravish R. Singh	Laplace transform, properties of Laplace Transform, Differential Equations (1^{st} & 2^{nd} Order)
		2L	Solution of problems using Laplace Transform			

					(Mcgraw hill Publications)	
Module-VII Two port network analysis Page (60 - 83)	**8L**	4L	Open circuit Impedance & Short circuit Admittance parameter, Transmission parameter, Hybrid Parameter	Two port network parameter calculation	1. Circuit Theory & Network Analysis by A. Chakraborti (Dhanpat Rai Publications) 2. Network Analysis & Synthesis by Ravish R. Singh (Mcgraw hill Publications)	Concept of n port network, T section, pi section.
		4L	Conditions Of Reciprocity And Symmetry, Interrelation between different parameters, Driving point impedance & Admittance. Interconnection Of Two Port Networks. Solution of problems			

RESONANCE

1.1 Introduction

- An A.C. circuit is said to be in resonance when the applied voltage and current are in same phase.
- Any passive circuit will resonate if it has inductor and capacitor
- Resonance used in electronics circuit to select or tune a specific frequency signal
- In resonance condition as circuit current and applied voltage are in phase therefore power factor of the circuit is one.
- Alternately at resonance condition impedance offered by the circuit is purely resistive i.e. reactance part of the circuit is zero at resonance.

1.2 Types of Resonance Circuit

There are two types of resonance circuit (i) Series resonance circuit (ii) Parallel resonance circuit

1.2.1 Series Resonance Circuit

The circuit diagram of series resonance circuit is shown in Fig. 1

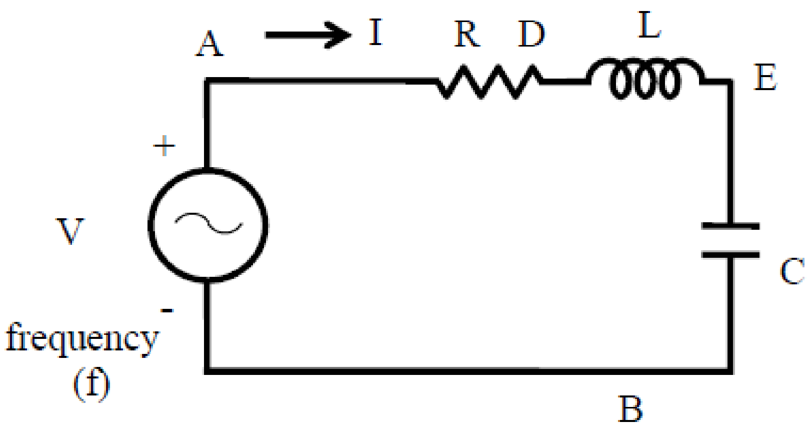


Fig.1 : Series resonance circuit [Courtesy – NPTEL lecture]

The input impedance is given by

$$Z = R + j(wL - \frac{1}{wC})$$

The current through the circuit is given by

$$I = \frac{V}{\sqrt{R^2 + (wL - \frac{1}{wC})^2}}$$

At resonance circuit must have unity power factor . That is reactive part is zero. Which leads

$$wL = \frac{1}{wC}$$

Alternately

$$Z\angle\phi = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

Where,

$$Z = \sqrt{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]} \;;\quad \phi = \tan^{-1}\frac{(\omega L - 1/\omega C)}{R} \;;\quad \omega = 2\pi f$$

The current is

$$I\angle-\phi = \frac{V\angle 0^\circ}{Z\angle\phi} = (V/Z)\angle-\phi$$

Where ,

$$I = \frac{V}{\left[R^2 + (\omega L - (1/\omega C)^2\right]^{\frac{1}{2}}}$$

The current in the circuit is maximum

If

$$wL = \frac{1}{wC}$$

At resonance condition maximum current flows through circuit .

Current at resonance is $I_0 = I_m = \frac{V}{R}$

Frequency at resonance

$$f_o = \frac{\omega_o}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

- The plotting of impedance and reactance in series resonance circuit is shown in Fig 2.

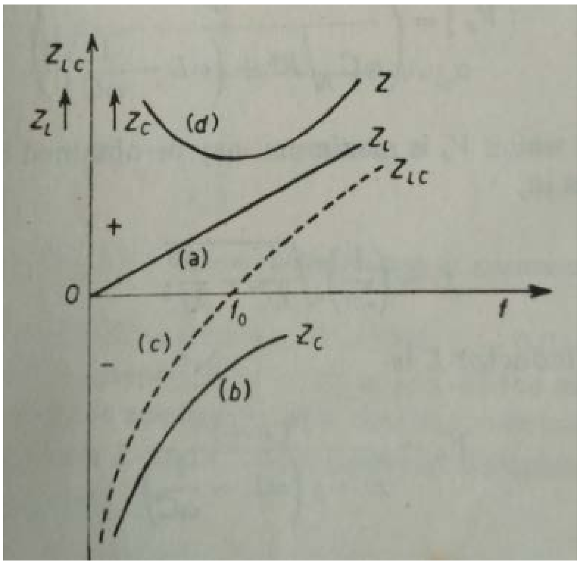


Fig. 2 Variation of impedance and reactance at series resonance circuit [Courtesy –Network and System , D.Roy Chowdhury]

- The variation of current and voltage and phasor diagram at series resonance circuit is shown in Fig. 3and Fig. 4 respectively

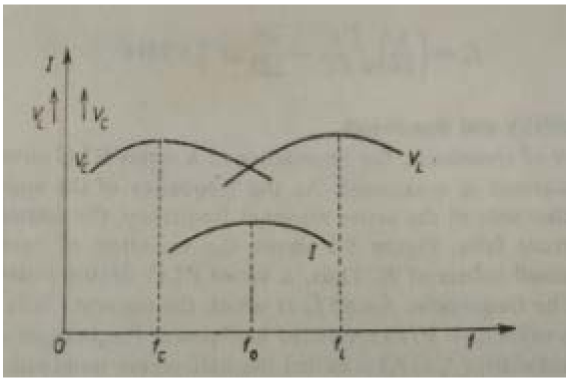


Fig. 3 : Current and voltage variation in series resonance circuit [Courtesy –Network and System , D.Roy Chowdhury]

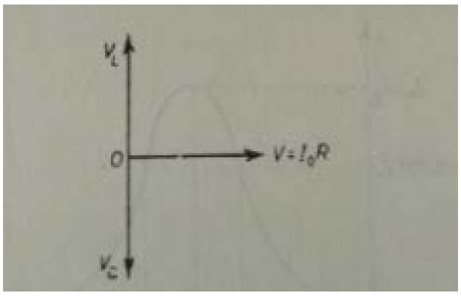


Fig 4: Phasor diagram at series resonance circuit [Courtesy –Network and System , D.Roy Chowdhury]

- Selectivity and bandwidth of series resonance circuit is discussed below
  The variation of current at series resonance circuit is shown in Fig. 5

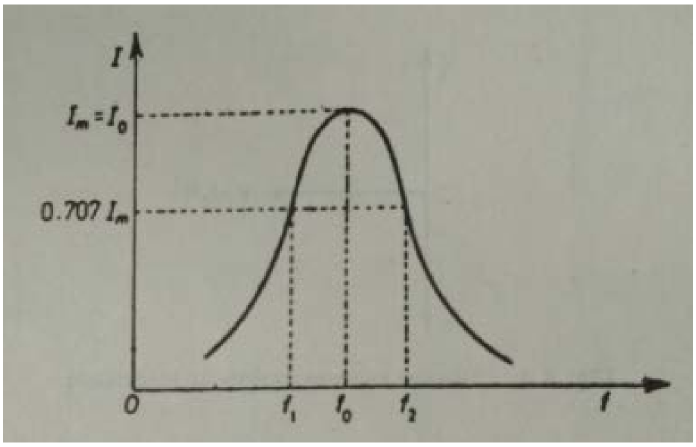


Fig. 5: Variation of current at series resonance circuit

The selectivity of the circuit is defined as [Courtesy –Network and System , D.Roy Chowdhury]

$$Selectivity = \frac{Resonance\ frequency}{Bandwidth} = \frac{f_0}{f_2 - f_1}$$

Where f_1 and f_2 are lower and upper cut-off frequencies respectively . These frequencies also known as half power frequency or -3dB frequency.

Bandwidth (B.W.) = $f_2 - f_1 = \frac{R}{L}$

- The variation of current with respect to frequency for different value of resistance is shown in Fig. 6

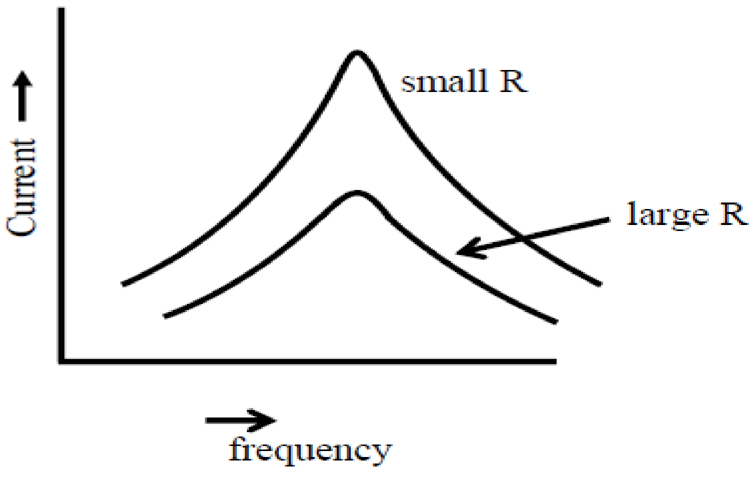


Fig. 6 : variation of current with respect to frequency for different value of resistance [Courtesy – NPTEL lecture]

- Quality factor or Q – factor of resonance circuit is expressed as

$$Q = 2\pi \left[\frac{Maximum\ Energy\ Stored\ Per\ Cycle}{Energy\ Dissipated\ Per\ Cycle}\right]$$

Q factor for inductor is given by $Q = \frac{\omega L}{R}$

Q factor for capacitor is given by $Q = \frac{1}{\omega CR}$

At resonance $Q_0 = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR} = \frac{1}{R} . \sqrt{\frac{L}{C}}$

1.2.2 Parallel or anti resonance Circuit

The circuit diagram of parallel resonance is shown in Fig.7

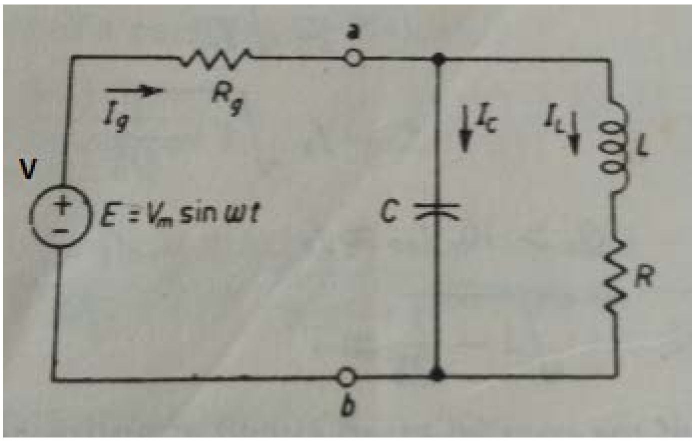


Fig. 7 : Parallel resonance circuit [Courtesy –Network and System , D.Roy Chowdhury]

- Resonance frequency

  The admittances in circuit 1.7 are $Y_L = \frac{1}{R+j\omega L}$, $Y_c = j\omega C$

  Total admittance of the circuit $Y = Y_L + Y_c = \frac{R}{R^2+\omega^2 L^2} - j\left(\frac{\omega L}{R^2+\omega^2 L^2} - \omega C\right)$

  At anti-resonance , the circuit must have unity power factor

  That is $\left(\frac{\omega_{ar} L}{R^2+\omega_{ar}{}^2 L^2} - \omega_{ar} C\right) = 0$

  or , $R^2 + \omega_{ar}{}^2 L^2 = \frac{L}{C}$

  Therefore $\omega_{ar} = \sqrt{\left(\frac{1}{LC} - \frac{R^2}{L^2}\right)}$ and $f_{ar} = \frac{1}{2\pi}\sqrt{\left(\frac{1}{LC} - \frac{R^2}{L^2}\right)} = f_0\sqrt{(1 - \frac{1}{Q_0{}^2})}$

  Where f_0 is the resonance frequency of series resonance circuit. Q_0 is the quality factor .

- The phasor representation of parallel resonance circuit is shown in Fig. 8

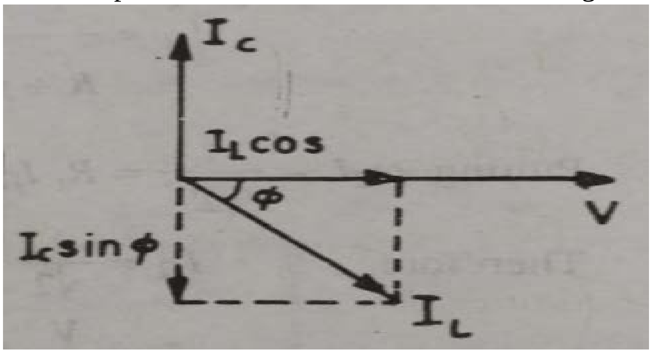


Fig. 8 : Phasor Representation of parallel resonance circuit [Courtesy – Network Filters and Transmission Line by A. Chakrabarti]

- The fig. 9 shows the variation of susceptance in parallel resonance circuit

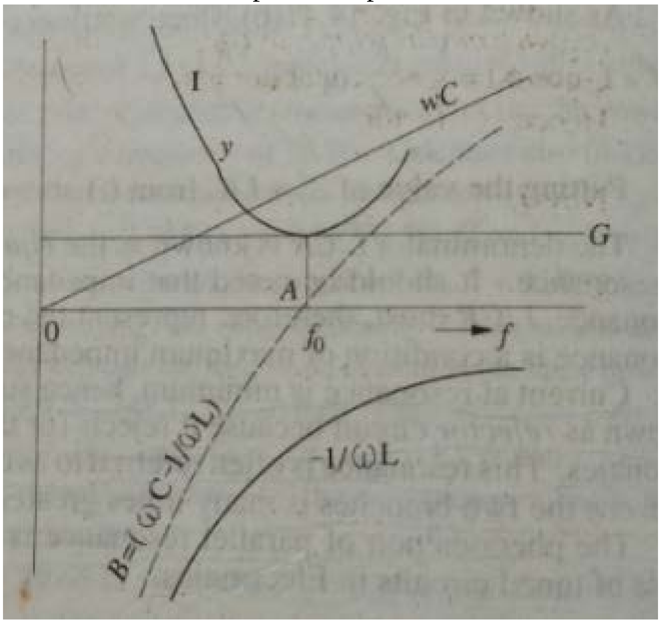


Fig. 9: Variation of susceptance with frequency [Courtesy – Electrical Technology by A.K and B.L. Thereja]

- **Selectivity and bandwidth**

  Selectivity can be measured in-terms of quality factor . Higher is the Q better is the selectivity . Q can be expressed as $Q = \frac{f_{ar}}{B.W.}$

  The quality factor of anti-resonance circuit can also be expressed as

  $Q = \frac{\omega_{ar} L}{R} = \frac{1}{\omega_{ar} CR} = \frac{1}{R}\sqrt{\frac{L}{C}}$

1.3 R-L-C parallel resonance circuit

The circuit diagram of R-L-C parallel resonance circuit is shown in Fig. 10

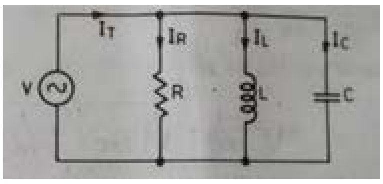


Fig. 10: circuit diagram of R-L-C parallel resonance circuit [Courtesy – Network Filters and Transmission Line by A. Chakrabarti]

The admittance of the circuit Fig. 1.10 is

$$Y = G + jB = \frac{1}{R} + j\omega C + \frac{1}{j\omega L} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

For resonance condition the susceptance part is zero

i.e. $\omega_n C - \frac{1}{\omega_n L} = 0$, *this leads* $\omega_n = \frac{1}{\sqrt{LC}}$

In parallel resonance circuit current becomes minimum .

- The variation of voltage and current of R-L-C parallel resonance circuit is shown in Fig. 11

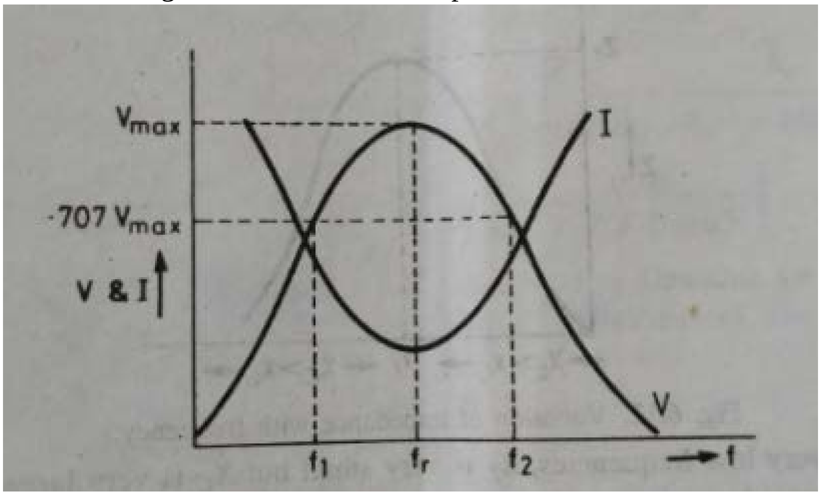


Fig. 11: Variation of voltage and current with frequency for R-L-C parallel resonance circuit [Courtesy – Network Filters and Transmission Line by A. Chakrabarti]

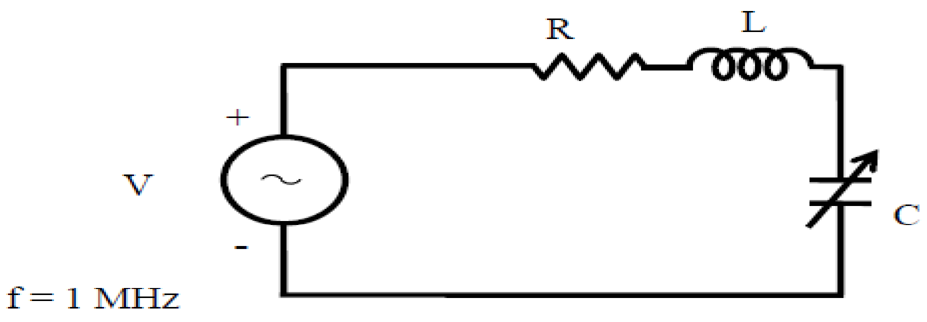
1.4 Assignment

Q.1.1 Draw the Laplace transform diagram of series and parallel resonance circuit. From the circuit derive the expression for resonance frequency in both cases . (5+5)+(5+5)

Q1.2 Draw the circuit diagram of one practical resonance circuit . Derive the expression for resonance frequency of the circuit . 5+5

Q1.3 Make a comparison list of series and parallel resonance circuit. 5

Q1.4. A constant voltage of frequency, 1 MHz is applied to a lossy inductor (r in series with L), in series with a variable capacitor, C as shown in fig below . The current drawn is maximum, when C = 400 pF; while current is reduced to 70.7% of the above value, when C = 450 pF. Find the values of r and L. Calculate also the quality factor of the coil, and the bandwidth.



Q1.5 . Derive an expression for upper and lower cut-off frequency of series and parallel resonance curve . 5+5

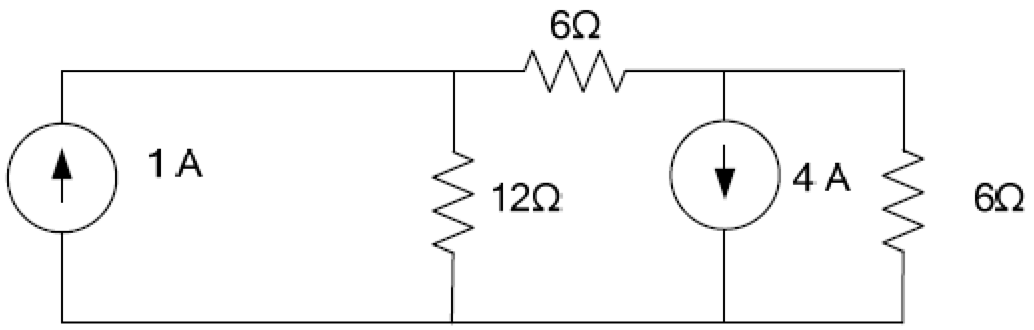
NETWORK ANALYSIS

2.1 Nodal Analysis

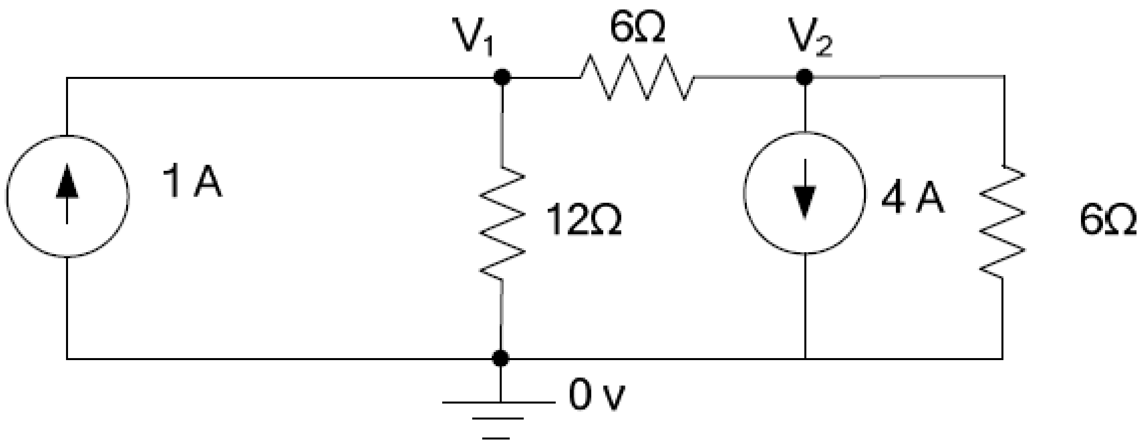
- Nodal analysis is a helpful technique to find out node voltage , branch voltage , branch current e.t.c.
- In this method as a 1st step reference node or datum node is selected on the circuit
- KCL equations are written at target node and adjacent node in-terms of node voltage with references to datum node
- During writing node equations, for all node it has to be considered only one : either current is leaving from target node towards adjacent node or current is coming towards target node from adjacent node .That is to be considered for all node in a circuit for nodal analysis
- Nodal analysis is illustrated in the following example

Case 1: Circuits with Independent Current and/voltage Sources

Example 1 : For the circuit shown in figure determine the node voltages



Sol^n : Identify node and reference node . Mention node voltages . voltage of reference node is set to be zero .Following is the implementation



Applying the nodal analysis at node 1 we get (Basically we are writing KCL equation at node 1)

$$-1 + \frac{V_1 - 0}{12} + \frac{V_1 - V_2}{6} = 0$$

$$3V_1 - 2V_2 = 12 \qquad (1)$$

(Please note here we have assumed voltage V_1 is greater than the voltages of adjacent node,i.e. current is leaving from node 1)

Similarly applying nodal analysis at node 2 we get

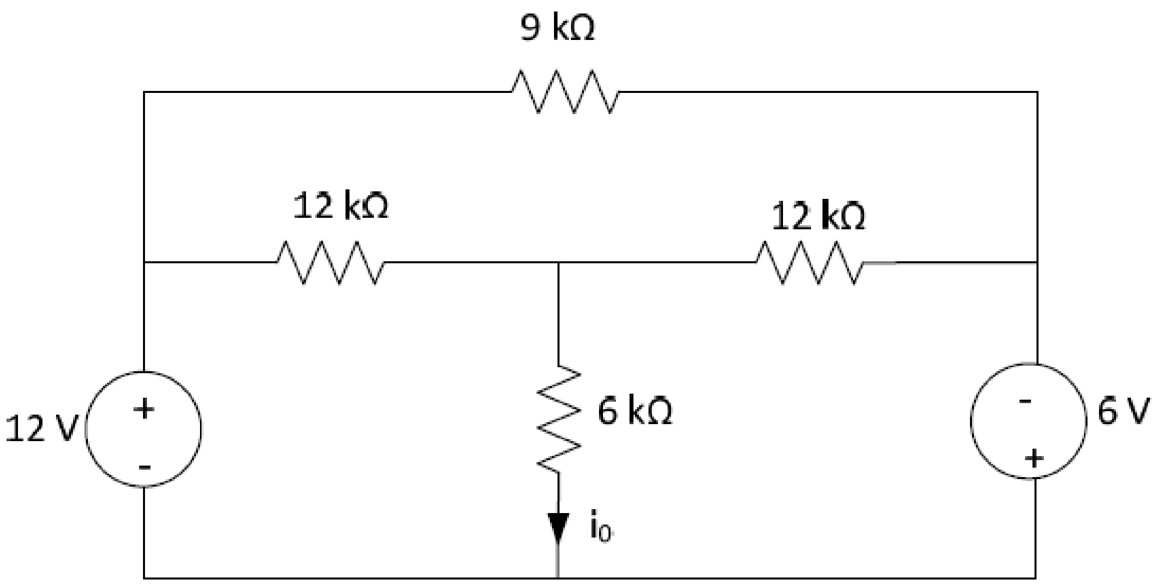
$$4 + \frac{V_2 - 0}{6} + \frac{V_2 - V_1}{6} = 0$$

$$-V_1 + 2V_2 = -24 \qquad (2)$$

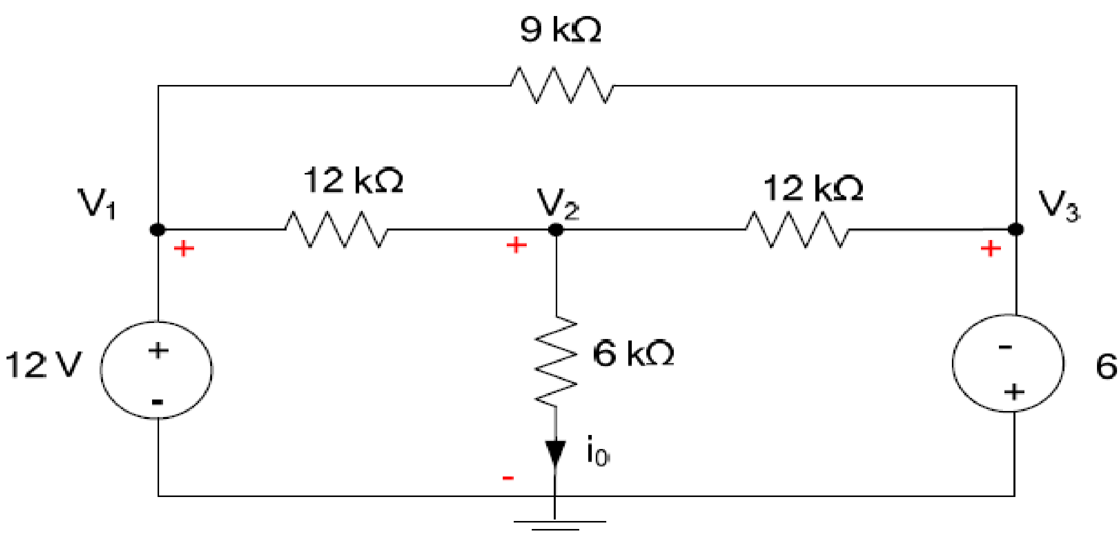
Solving equation 1 and 2 we get

$$V_1 = -6V$$
$$V_2 = -15\ V$$

Example 2 : Find the current I_0 using nodal analysis in the circuit below



Solution : Node voltages and reference node are identified in the following circuit



Here V_1 = 12 V , V_2 = -6 V

Writing node equation at node 2 , we get

$$\frac{V_2}{6\,k} + \frac{V_2 - V_1}{12\,k} + \frac{V_2 - V_3}{12\,k} = 0$$

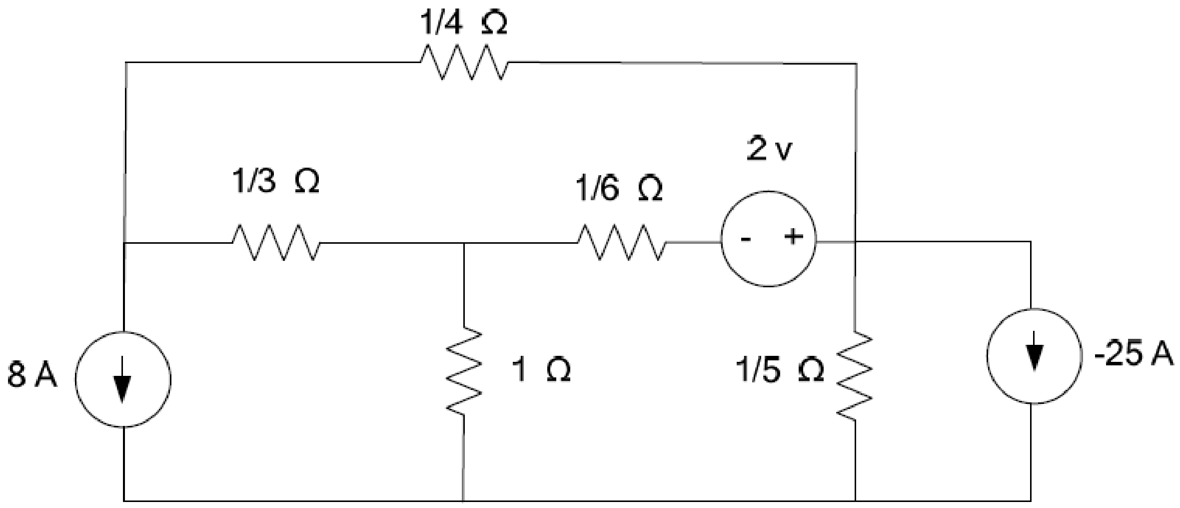
=>

$$\frac{V_2}{6\,k} + \frac{V_2 - 12}{12\,k} + \frac{V_2 - (-6)}{12\,k} = 0$$

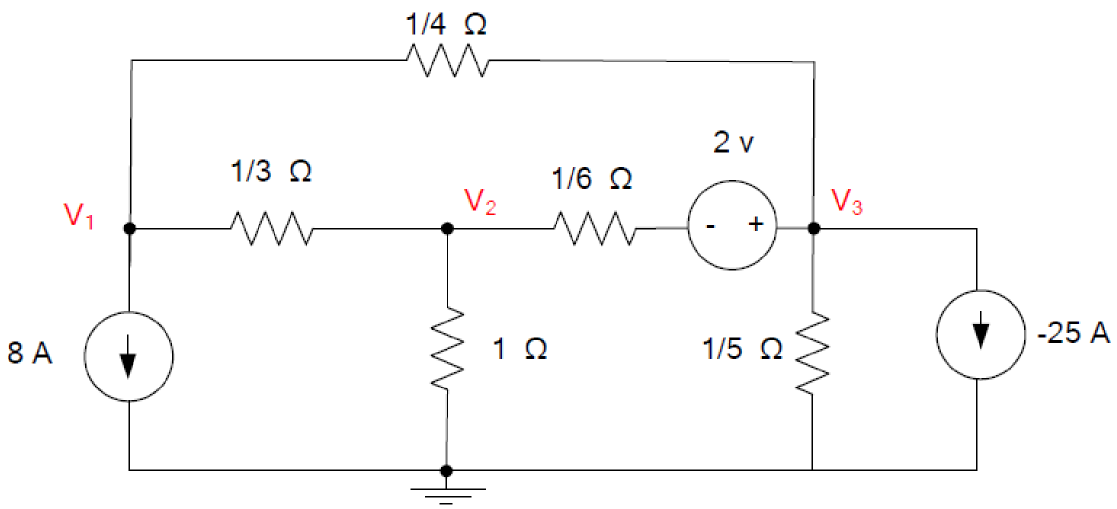
Solving we get V_2 = (3/2) V

Therefore current I_0 = (V_2 /6 K)

Example 3 : Write down the node equation of the following circuit



Solution :



Applying nodal analysis at node 3 we get

$$-25+\frac{V_3}{\frac{1}{5}}+\frac{V_3-V_1}{\frac{1}{4}}+\frac{(V_3-2)-V_2}{\frac{1}{6}}=0$$

$$-4\,V_1-6\,V_2+15\,V_3=37 \qquad (1)$$

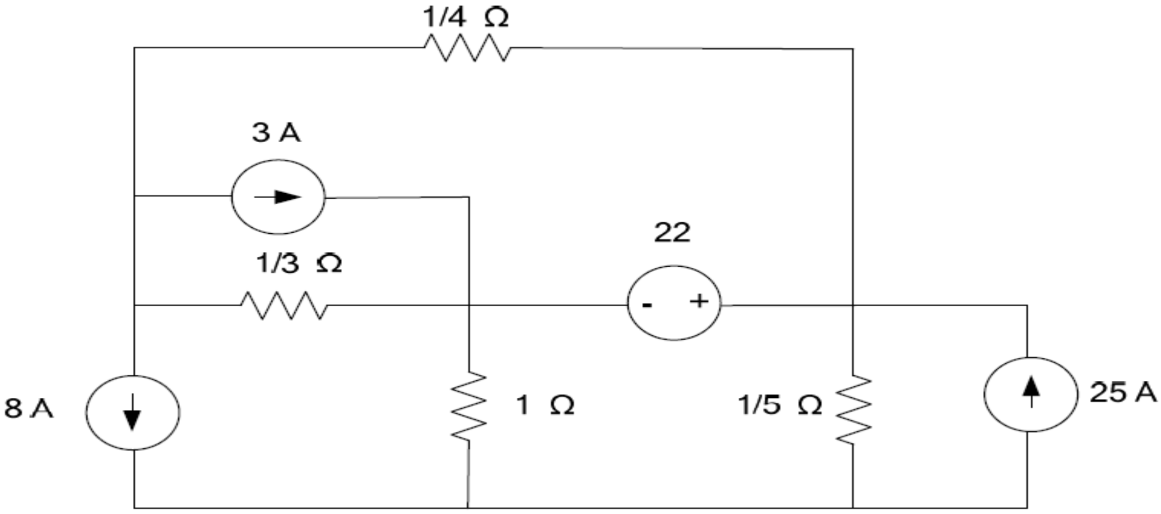
Nodal equation at node 2,
$$\frac{V_2}{1}+\frac{V_2-V_1}{\frac{1}{3}}+\frac{V_2-(V_3-2)}{\frac{1}{6}}=0$$

$$-3\,V_1+10V_2-6\,V_3=-12 \qquad (2)$$

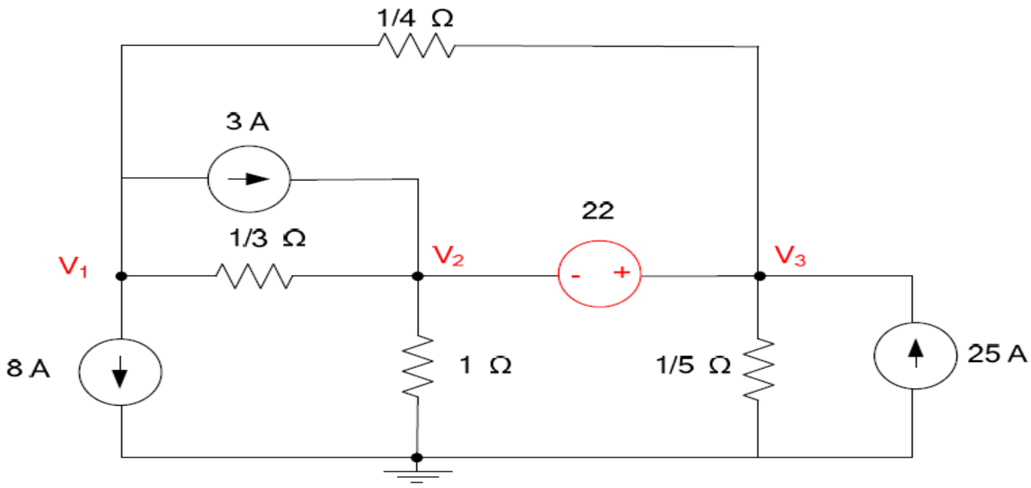
Nodal equation at node 1
$$8+\frac{V_1-V_3}{\frac{1}{4}}+\frac{V_1-V_2}{\frac{1}{3}}=0$$

$$7\,V_1-3\,V_2-4\,V_3=-8 \qquad (3)$$

Example 4 : Find the node voltages for the following circuit using nodal analysis



Solution: The circuit can be redrawn with node name as follows

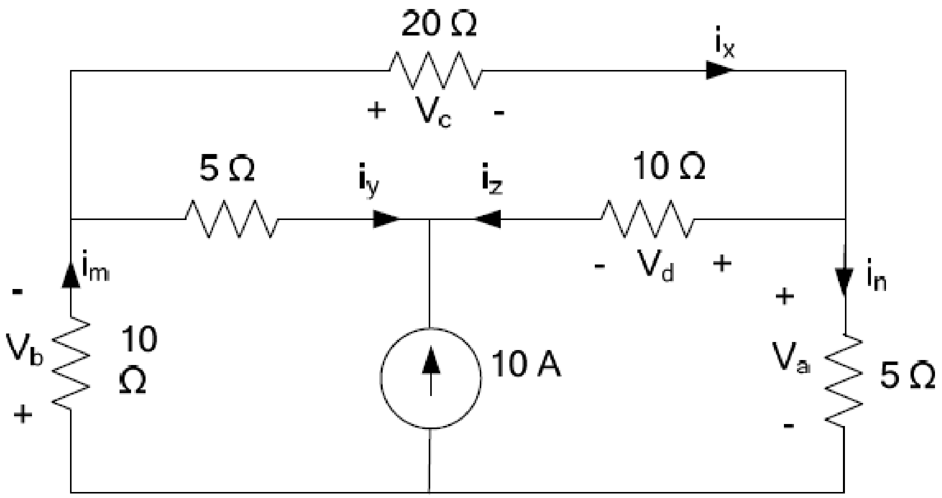


Hints : This is note a conventional nodal analysis problem . In addition with conventional nodal analysis at node V_1 , node equation for V_2 and V_3 can be carried out considering supernode consisting of node V_2 and V_3 and associated branch .

2.2 Assignment

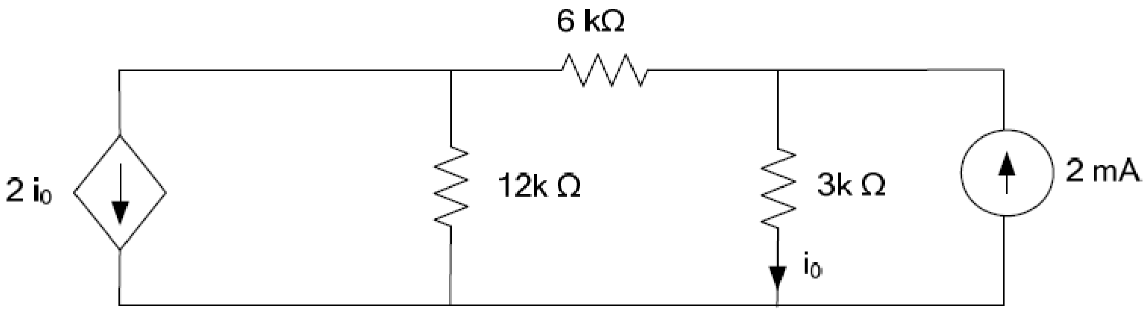
Q1.

Use the nodal analysis to find the i_x, i_y, i_z, i_n, i_m, V_a, V_b, V_c, V_d .



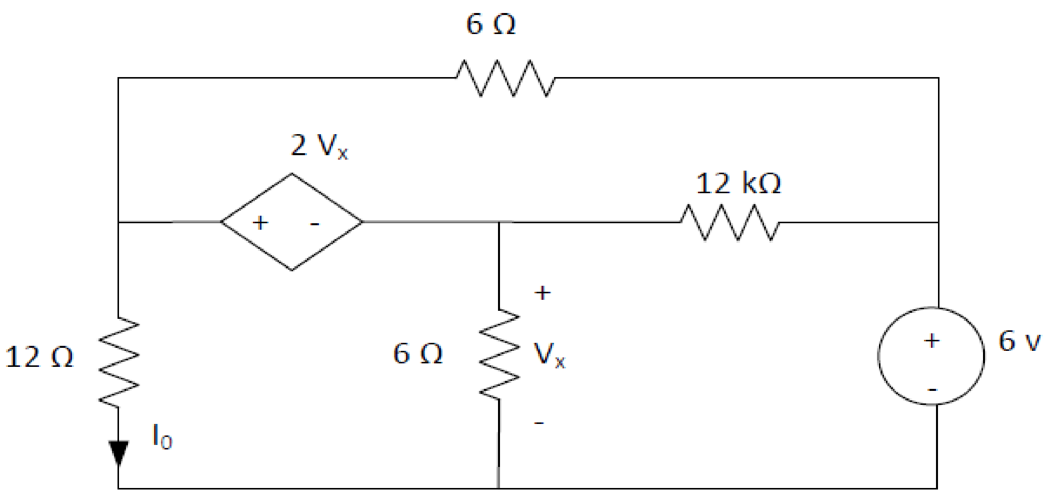
Q2.

Find the node voltages for the following:



Q3.

Find the current I_0 by using the nodal analysis.



2.3 Mesh Analysis

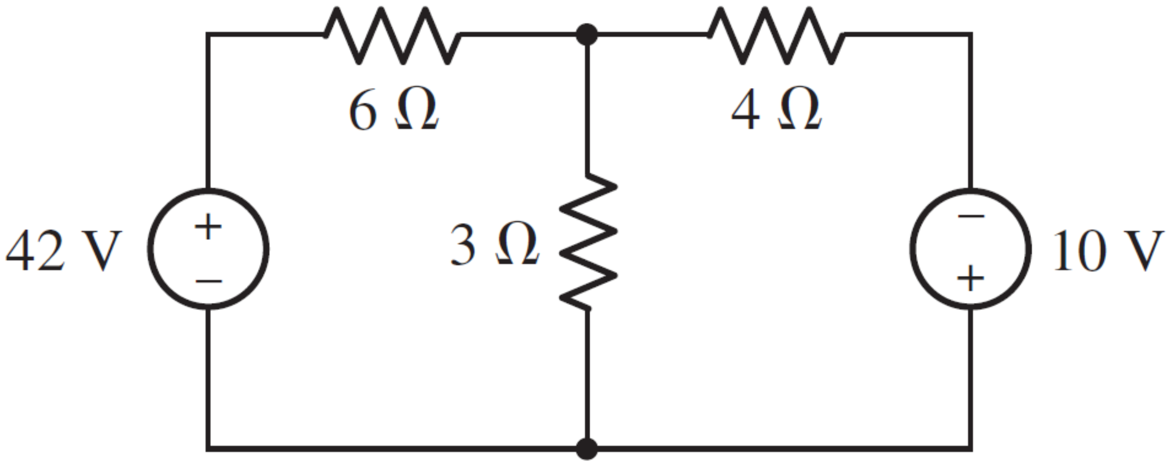
Loop : A loop is a closed path that is drawn by starting at anode and tracing a path back to that node without passing through an intermediate node more than once

Mesh : A Mesh is a loop that contains no other loops within it

We use Mesh Current analysis when we have circuits: that contain multiple sources and we wish to solve for currents.

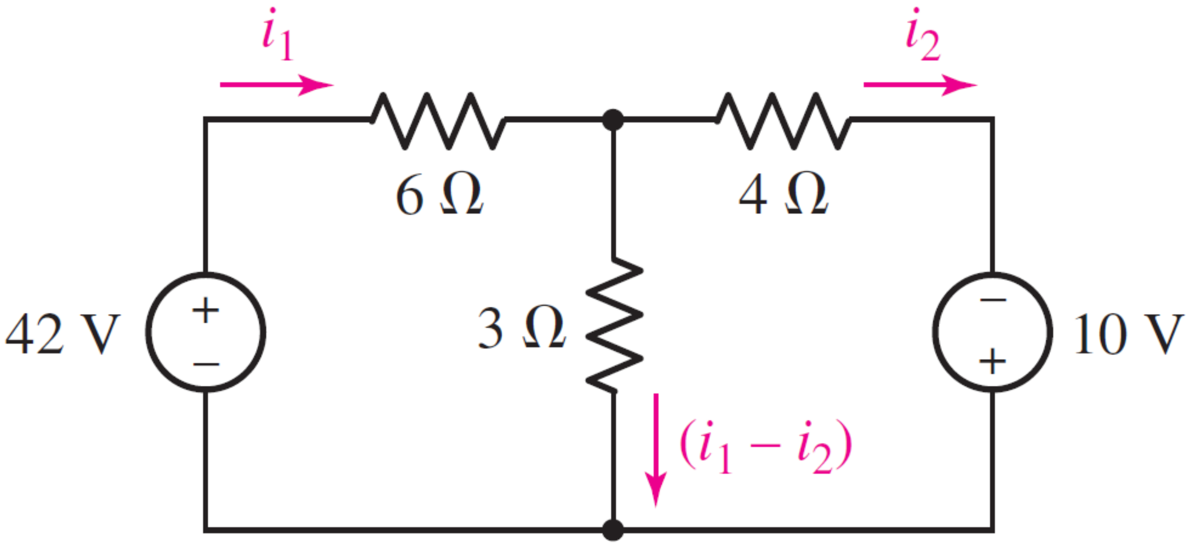
Using following examples Mesh analysis has been illustrated

Example 1: For the circuit shown in figure determine current delivered by the sources



Solution :

Label each of the M mesh currents (defining all mesh currents to flow clockwise results in a simpler analysis) .Write a KVL equation around each mesh .



For mesh 1, we have

$$-42 + 6i_1 + 3(i_1 - i_2) = 0$$

or

$$9i_1 - 3i_2 = 42 \qquad (1)$$

For mesh 2 ,we have

$$-3(i_1 - i_2) + 4i_2 - 10 = 0$$

or

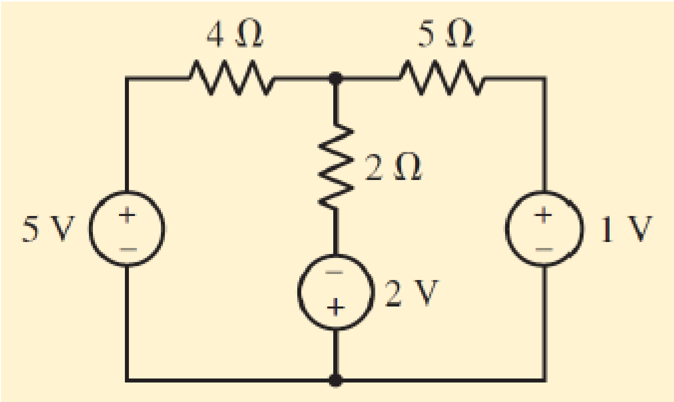
$$-3i_1 + 7i_2 = 10 \qquad (2)$$

Solving we get

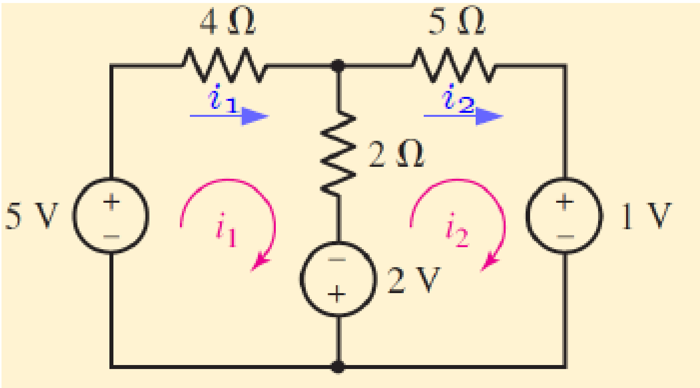
$$i_1 = 6\text{ A} \qquad i_2 = 4\text{ A}$$

(N.B. Please note if number of mesh current equations is more then then solution of equation can be carried out using matrix method – Cramers rules)

Example 2 : Determine power supplied by 2 Volt source in the figure below.



Solution :



Mesh equations are

$$-5 + 4i_1 + 2(i_1 - i_2) - 2 = 0$$

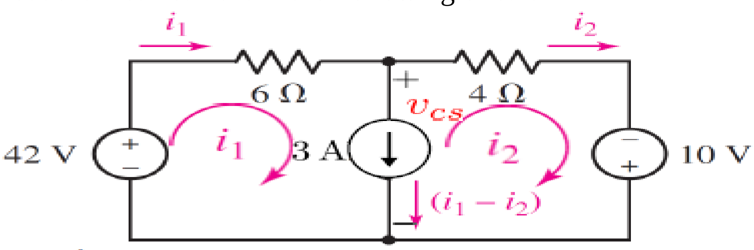
$$+2 - 2(i_1 - i_2) + 5i_2 + 1 = 0$$

Solving above two equations we get

i_1=1.132 A and i_2=-0.1053 A

Therefore power delivered by the 2 Volt source is =2($i_1 - i_2$) = 2.4 W

Example 3 : Find out the mesh current in the following circuit



Solution : The circuit can be analysed from the concept of supermesh

A supermesh is formed when a current source is the only element connected between two meshes

a. Define a voltage across the source and write KVL equations for the two meshes

$$-42 + 6i_1 + v_{cs} = 0$$

and

$$v_{cs} = 4i_2 - 10$$

b. We do not need to evaluate v_{cs} to solve the circuit.

We get

$$-42 + 6i_1 + 4i_2 - 10 = 0$$

c. Finally, the source current is related to the mesh currents

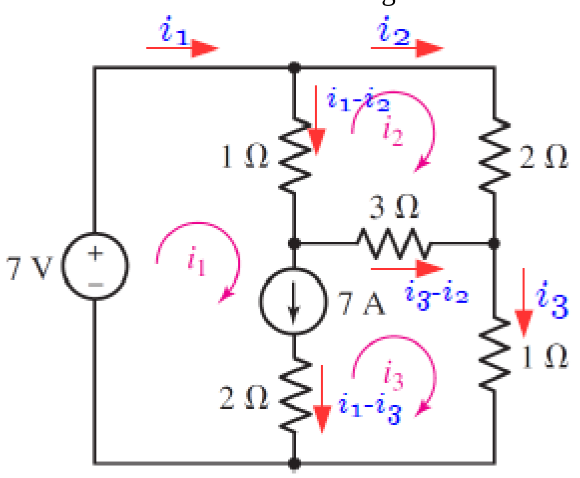
That is

$$i_1 - i_2 = 3$$

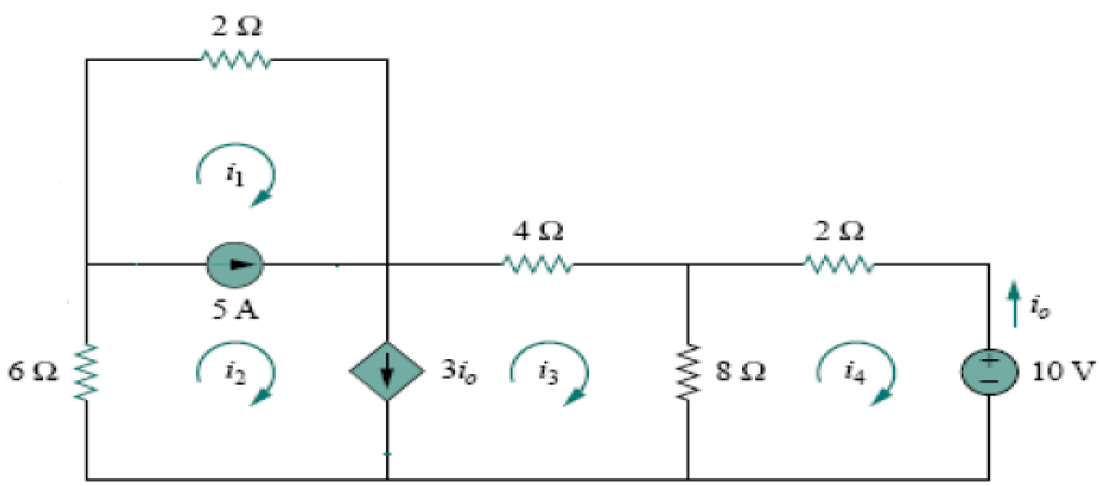
d. The equation from b and c can be solved to get mesh currents

2.4 Assignments

Q1. Determine the three mesh currents of the following circuit



Q2. Use the mesh current method to determine current i_0



2.5 Network Theorems:

Superposition Theorem:

The total current in any part of a linear circuit equals the algebraic sum of the currents produced by each source separately. To evaluate the separate currents to be combined, replace all other ideal voltage sources by short circuits and all other ideal current sources by open circuits. In practical case, replace all other voltage sources by internal series resistance in circuits and all other current sources by internal parallel resistance circuits.

When multiple sources are used in a circuit, the Superposition theorem provides a method for analysis.

The steps in applying the superposition method are as follows:

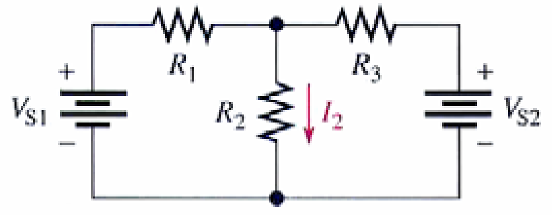
Step 1. Leave one voltage (or current) source at a time in the circuit and replace each of the other voltage (or current) sources with its internal resistance. For ideal sources a short represents zero internal resistance and an open represents infinite internal resistance.

Step 2. Determine the particular current (or voltage) that you want just as if there were only one source in the circuit.

Step 3. Take the next source in the circuit and repeat Steps 1 and 2. Do this for each source.

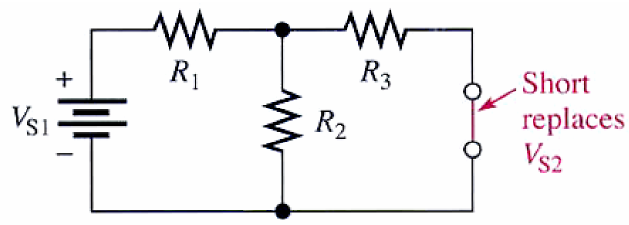
Step 4. To find the actual current in a given branch, algebraically sum the currents due to each individual source. (If the currents are in the same direction, they are added. If the currents are in opposite directions, they are subtracted with the direction of the resulting current the same as the larger of the original quantities.) Once you find the current, you can determine the voltage using Ohm's law.

The approch to superposition is demonstrated in the figure for a serias-parallel circuit with two ideal voltage sources. We take that figure (a) is a given ciruit.

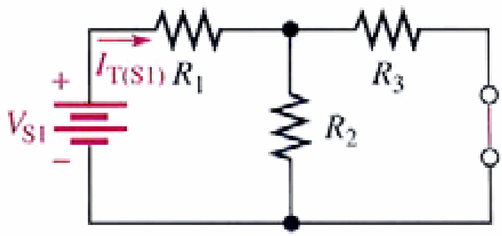


(a) Problem: Find I_2.

- Analysis current for only voltage source V_{S1}: figure (b),(c) and (d)



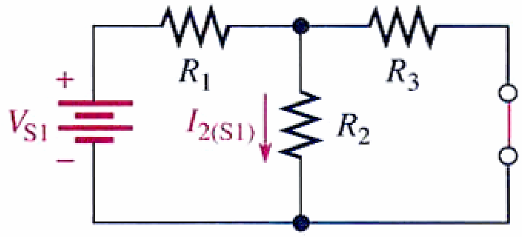
(b) Replace V_{S2} with zero resistance (short).



(c) Find R_T and I_T looking from V_{S1}:

$R_{T(S1)} = R_1 + R_2 \parallel R_3$

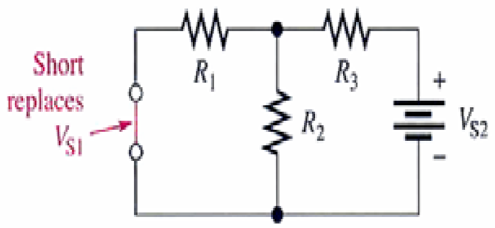
$I_{T(S1)} = V_{S1}/R_{T(S1)}$



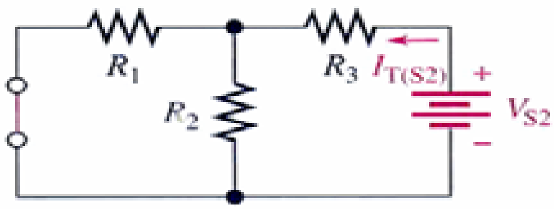
(d) Find I_2 due to V_{S1} (current divider):

$$I_{2(S1)} = \left(\frac{R_3}{R_2 + R_3}\right) I_{T(S1)}$$

- Analysis current for only voltage source V_{S2}: figure (e),(f)and (g)



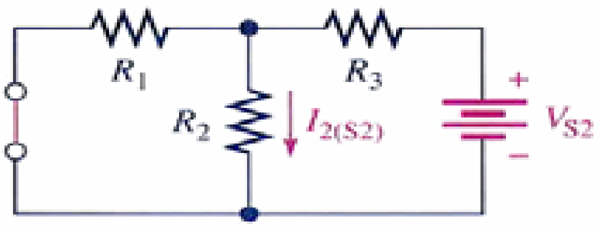
(e) Replace V_{S1} with zero resistance (short).



(f) Find R_T and I_T looking from V_{S2}:

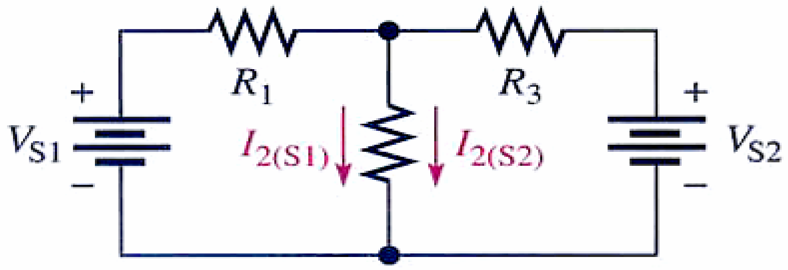
$R_{T(S2)} = R_3 + R_1 \parallel R_2$

$I_{T(S2)} = V_{S2}/R_{T(S2)}$



(g) Find I_2 due to V_{S2}:

$$I_{2(S2)} = \left(\frac{R_1}{R_1 + R_2}\right) I_{T(S2)}$$

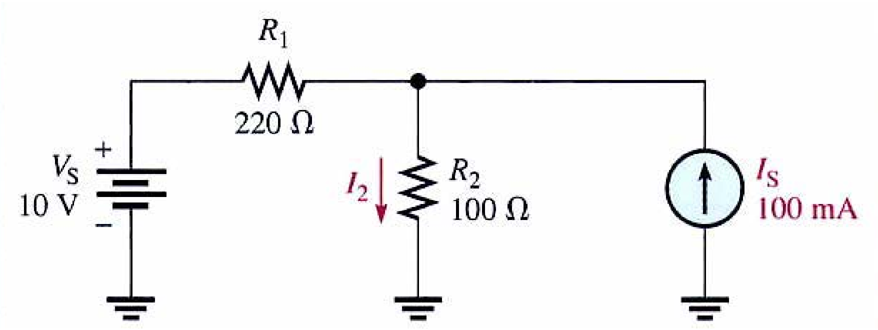


(h) Restore the original sources. Add $I_{2(S1)}$ and $I_{2(S2)}$ to get the actual I_2 (they are in same direction):

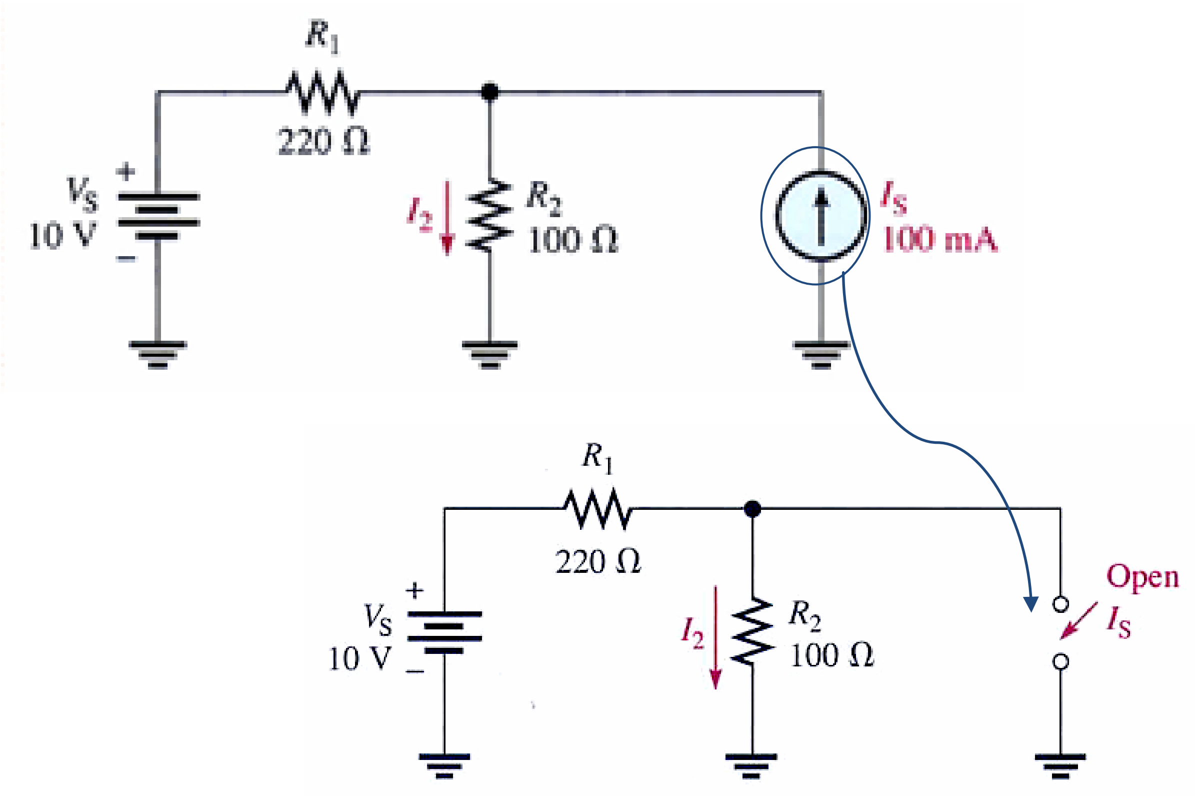
$$I_2 = I_{2(S1)} + I_{2(S2)}$$

Figure (h) is the final result.

Example 1: Find the current through R_2 in the circuit.



Solution:



Step1: Find the current through R2 due to Vs by replacing I_s with an open.

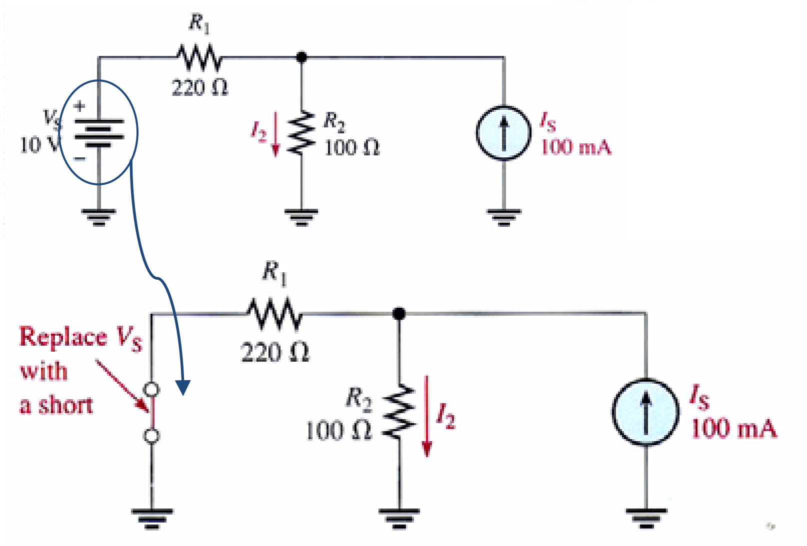
Notice that all of the current produced by Vs is through R_2. Looking from Vs ,

$R_T=R_1+R_2=320\Omega$

The current thought R_2 due to Vs is

$I_{2(Vs)}=Vs/R_T= 10V/320\Omega=31.2mA$

Note this current is downward through R_2.



Step2: Find the current through R_2 due to Is by replacing Vs with a short,

Use the current-divider formula to determine the current through R_2 due to Is,

$$I_{2(I_S)} = \left(\frac{R_1}{R_1 + R_2}\right) I_S = \left(\frac{220\Omega}{320\Omega}\right) 100mA = 68.8mA$$

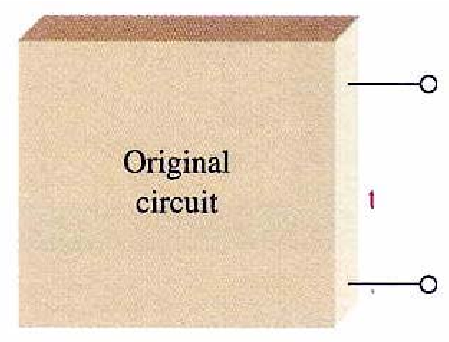
Note this current is downward through R_2.

Step3: Both currents are in the same direction through R2, so add them to the total.

$$I_{2(TOTAL)} = I_{2(I_S)} + I_{2(V_S)} = 68.8mA + 31.2mA = 100mA$$

Thevenin's theorem

In electrical circuit theory, Thevenin's theorem for linear electrical networks states that any combination of voltage sources, current sources and resistors with two terminals is electrically equivalent to a single voltage source V and a single series resistor R. For single frequency AC systems, the theorem can also be applied to general impedances, not just resistors. Any complex network can be reduced to a Thevenin's equivalent circuit consist of a single voltage source (V_{TH}) and series resistance (R_{TH}) connected to a load.



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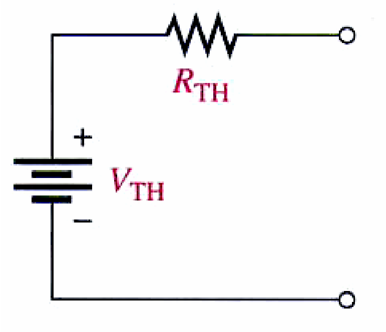
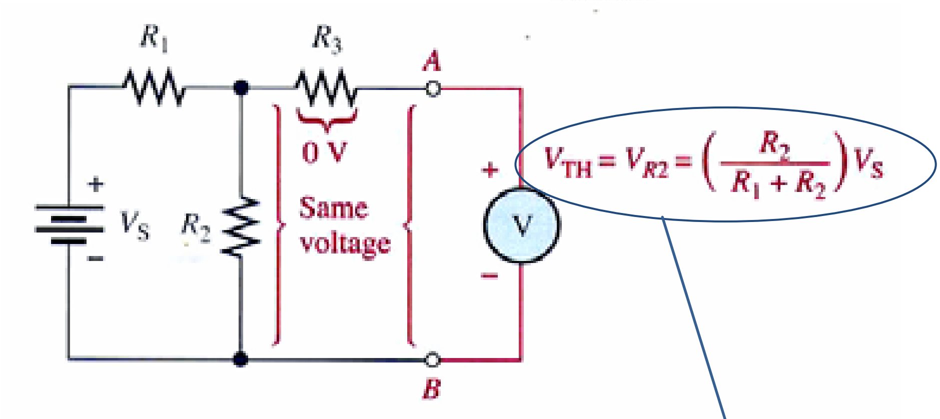


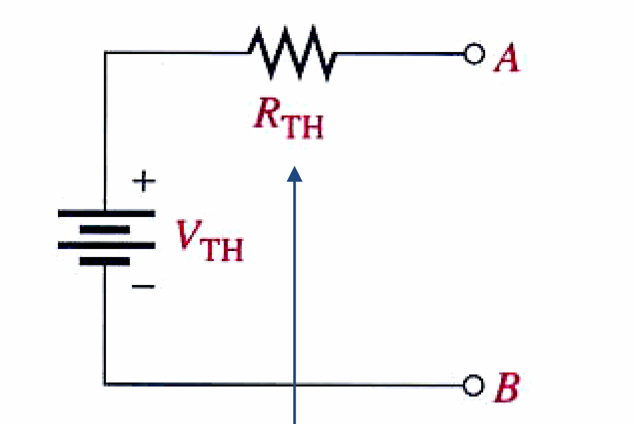
The steps of the simplification of a circuit by Thevenin's theorem as follows:

step1: The Thevenin's equivalent voltage (V_{TH}) is open circuit (no-load) voltage between two terminals 'A' &'B' in a circuit in figure (a).

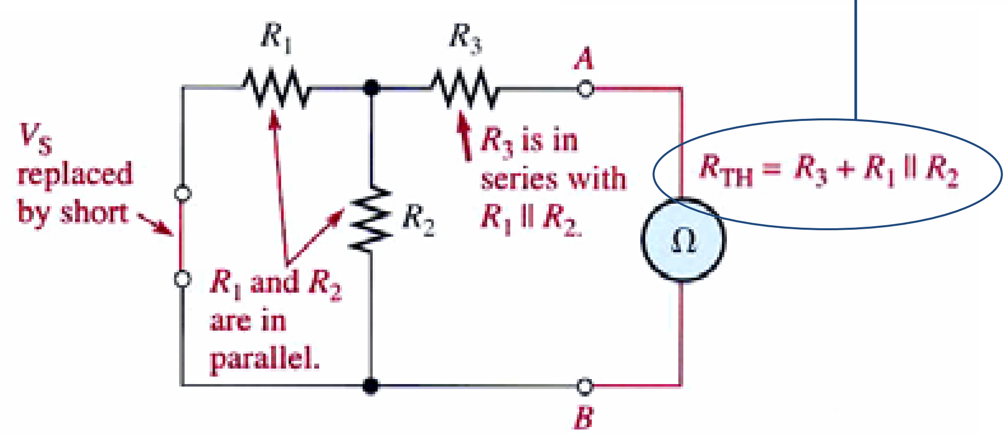


(a) Finding V_{TH}

Step2: The Thevenin's equivalent resistance (R_{TH}) is the total resistance appearing between two terminals 'A' &'B' in given circuit (figure (b)) with all sources replaced by their internal resistance.



(c) Thevenin equivalent circuit

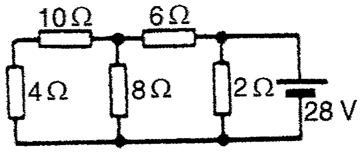


(b) Finding R_{TH}

Step3: Finaly found figure (c) The Thevenin's equivalent circuit.

Example 1:

In the network shown below, the battery has negligible internal resistance. Find, using Thevenin's theorem, the current flowing in the 4 Ω resistor.



Solution:

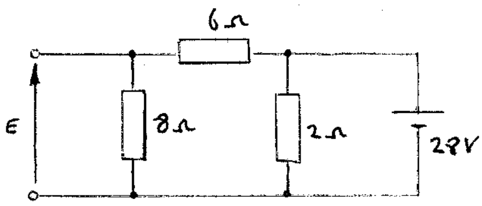


Fig.(a)

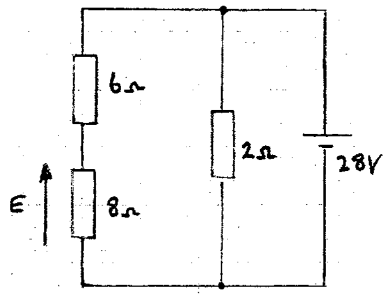


Fig.(b)

1. The resistors in the branch containing the 4 Ω resistor are removed as shown in diagram (a). Diagram (b) is diagram (a) redrawn.
2. By voltage division, open circuit e.m.f., E = $\left(\frac{8}{6+8}\right)(28)$ = 16 V

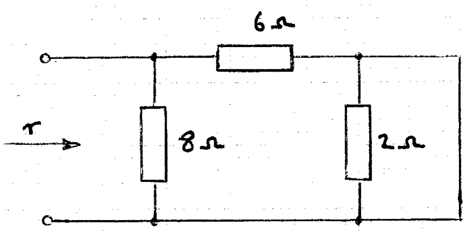


Fig.(c)

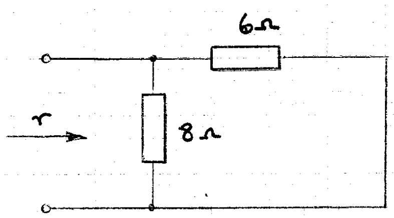


Fig.(d)

3. Replacing the 28 V source with a short circuit, the resistance r 'looking in' at the break is shown in diagram (c). The equivalent circuit of (c) is shown in (d), where r = $\frac{8 \times 6}{8+6} = \frac{48}{14}$ = 3.429 Ω

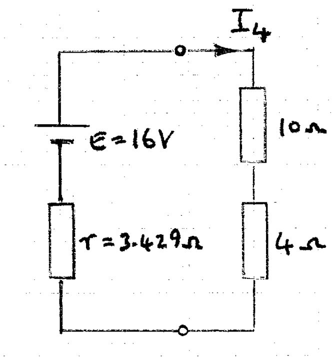


Fig.(e)

4. The Thevenin equivalent circuit is shown in diagram (e) where

current in 4 Ω resistor, $I_4 = \frac{16}{3.429+10+4}$ = **0.918 A**

Norton’s theorem

Any collection of batteries and resistances with two terminals is electrically equivalent to an ideal current source **I** in parallel with a single resistor $\mathbf{R_N}$. The value of $\mathbf{R_N}$ is the same as that in the Thevenin equivalent ($\mathbf{R_{TH}}$) and the current **I** can be found by dividing the open circuit voltage by $\mathbf{R_N}$.

The steps of the simplification of a circuit by Norton’s theorem as follows:

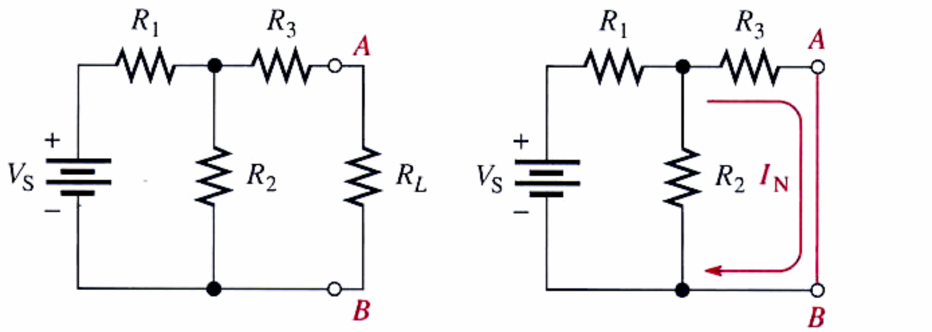
Step 1. Short the two terminals between which you want to find the Norton equivalent circuit.

Step 2. Determine the current (I_N) through the shorted terminals.

Step 3. Determine the resistance (R_N) between the two open terminals with all sources replaced with their internal resistances (ideal voltage sources shorted and ideal current sources opened). $R_N = R_{TH}$.

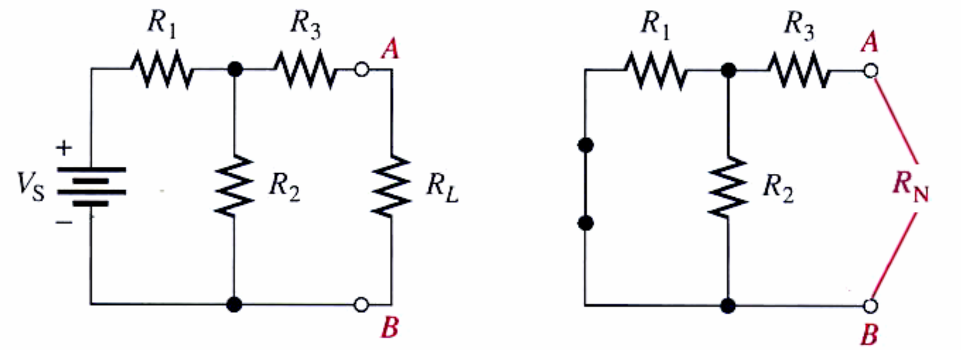
Step 4. Connect I_N and R_N in parallel to produce the complete Norton equivalent for the original circuit.

Norton’s equivalent current (I_N) is the short-circuit current between two output terminals in a circuit.

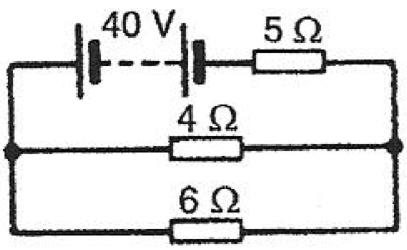


(a) Original circuit (b) Short the terminals to get I_N.

The Norton equivalent resistance, R_N, is the total resistance appearing between two output terminals in a given circuit with all sources replaced by their internal resistances.



Example 1: Use Norton's theorem to find the current flowing in the 6 Ω resistor shown below and the power dissipated in the 4 Ω resistor.



1. The branch containing the 6 Ω resistor is short circuited as shown in diagram (a).

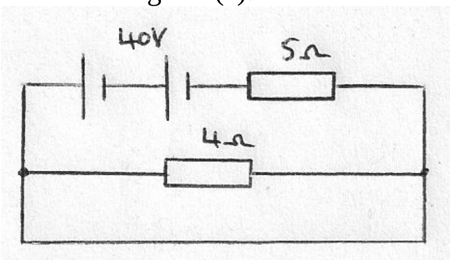


Fig.(a)

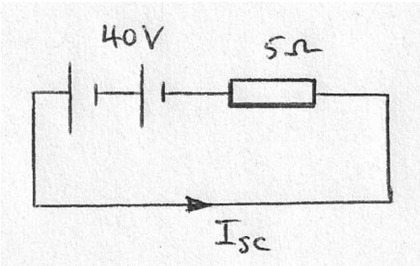


Fig.(b)

2. Diagram (b) is equivalent to diagram (a). From diagram (b), $I_{SC} = \frac{40}{5} = 8\ \text{A}$
3. With the voltage source removed, the resistance 'looking in' at a break in the short circuit is given by 4 Ω in parallel with 5 Ω, i.e. $r = \frac{4 \times 5}{4+5} = \frac{20}{9} = 2.2222\ \Omega$

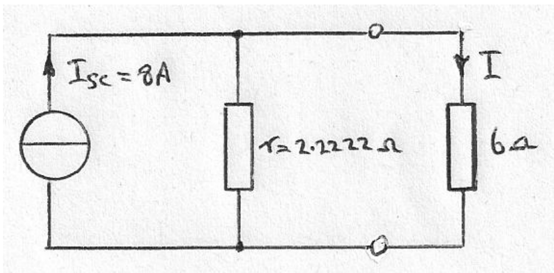


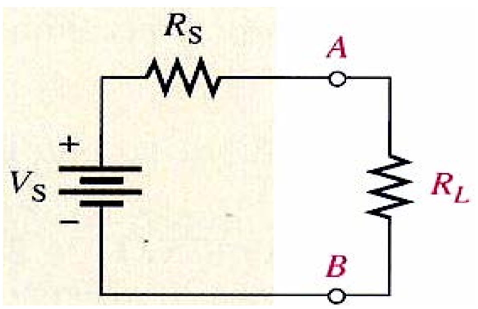
Fig.(c)

4. The Norton equivalent circuit is shown in diagram (c), where

the current in the 6 Ω resistor, $\mathbf{I_6} = \left(\frac{2.2222}{2.2222+6}\right)(8)$ **= 2.162 A**

Maximum Power Transfer Theorem

In electrical engineering, the **maximum power transfer theorem** states that, to obtain **maximum** external **power** from a source with a finite internal resistance, the resistance of the load must equal the resistance of the source as viewed from its output terminals.

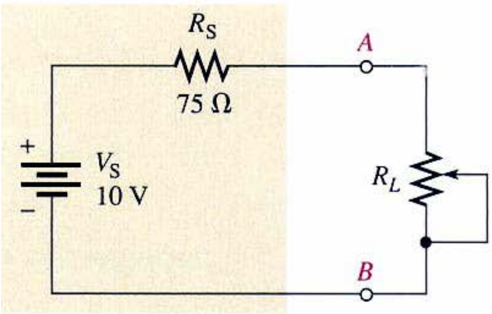


Maximum power is transferred to the load when R_L=R_S

Example 1: The source has an internal source resistance of 75Ω. Determine the load power for each of the following values of load resistance:

(i) 0Ω (ii) 25Ω (iii) 50Ω (iv) 75Ω (v) 100Ω (vi) 125Ω

Draw a graph showing the load power versus the load resistance.



Solution:

Use Ohm's law (I=V/R) and the power formula ($P=I^2R$) to find the load power, P_L, for each value of load resistance.

$$I=\frac{V_S}{R_S+R_L} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_(1)}$$

$$P_L=I^2R_L \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_(2)}$$

Using equation (1) & (2) for:

(i) R_L=0Ω $I=\frac{V_S}{R_S+R_L}=\frac{10V}{75\Omega+0\Omega}=133mA$ $P_L=I^2R_L=133^2\times 0=0mW$

(ii) R_L=25Ω $I=\frac{V_S}{R_S+R_L}=\frac{10V}{75\Omega+25\Omega}=100mA$ $P_L=I^2R_L=100^2\times 25=250mW$

(iii) R_L=50Ω $I=\frac{V_S}{R_S+R_L}=\frac{10V}{75\Omega+50\Omega}=80mA$ $P_L=I^2R_L=80^2\times 50=320mW$

(iv) R_L=75Ω $I=\frac{V_S}{R_S+R_L}=\frac{10V}{75\Omega+75\Omega}=66.7mA$ $P_L=I^2R_L=66.7^2\times 75=334mW$

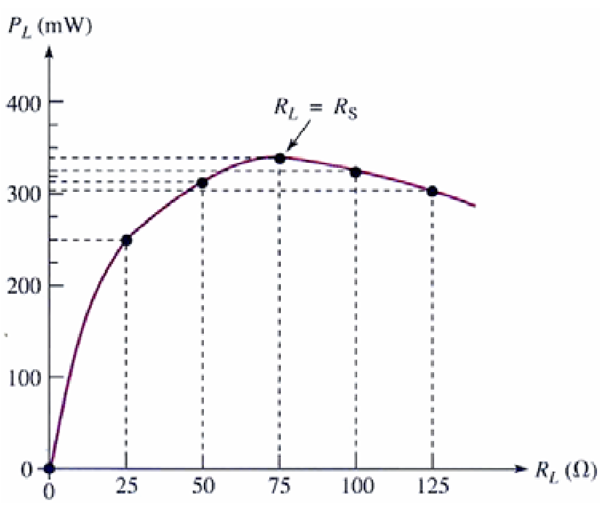
(v) R_L=100Ω $I=\frac{V_S}{R_S+R_L}=\frac{10V}{75\Omega+100\Omega}=57.1mA$

$P_L=I^2R_L=57.1^2\times 100=326mW$

(vi) R_L=125Ω

$I=\frac{V_S}{R_S+R_L}=\frac{10V}{75\Omega+125\Omega}=50mA$

$P_L=I^2R_L=50^2\times 125=313mW$



The load power is greatest when RL=75Ω, which is the same the internal source resistance.

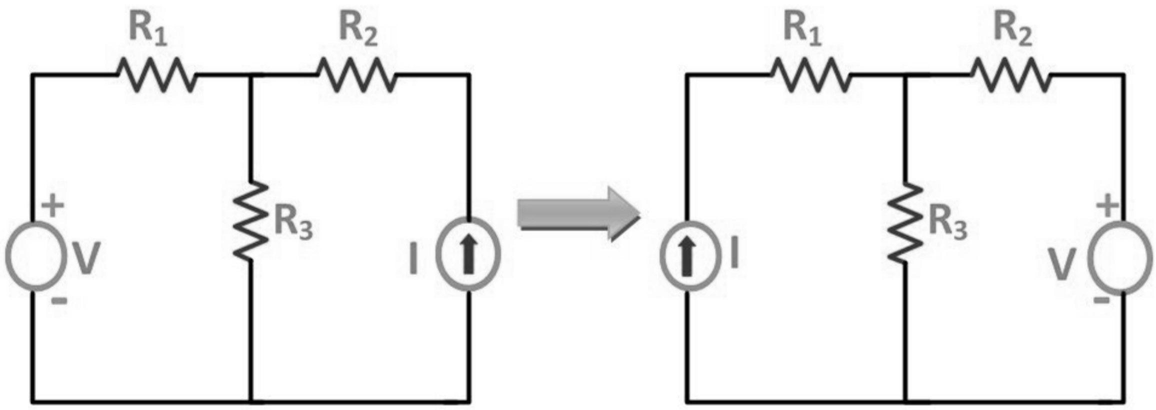
Reciprocity Theorem

Reciprocity Theorem states that – In any branch of a network or circuit, the current due to a single source of voltage (V) in the network is equal to the current through that branch in which the source was originally placed when the source is again put in the branch in which the current was originally obtained. This theorem is used in the bilateral linear network which is consists bilateral components.

Explanation of Reciprocity Theorem

The location of the voltage source and the current source may be interchanged without a change in current. However, the polarity of the voltage source should be identical with the direction of the branch current in each position.

The Reciprocity Theorem is explained with the help of the circuit diagram shown below



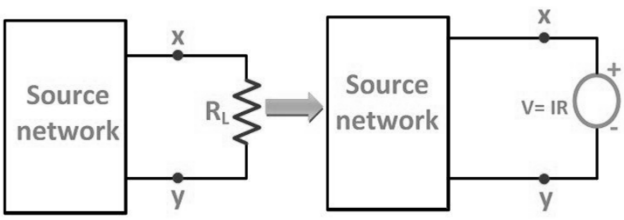
The various resistances R_1, R_2, R_3 is connected in the circuit diagram above with a voltage source (V) and a current source (I). It is clear from the figure above that the voltage source and current sources are interchanged for solving the network with the help of Reciprocity Theorem.

The limitation of this theorem is that it is applicable only to single source networks and not in the multi-source network. The network where reciprocity theorem is applied should be linear and consist of resistors, inductors, capacitors and coupled circuits. The circuit should not have any time-varying elements.

Compensation theorem

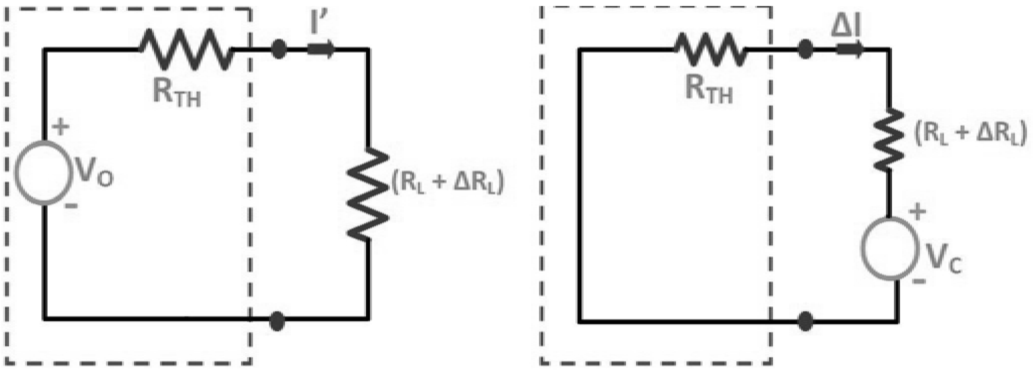
Compensation Theorem states that in a linear time invariant network when the resistance (R) of an uncoupled branch, carrying a current (I), is changed by (ΔR). The currents in all the branches would change and can be obtained by assuming that an ideal voltage source of (V_C) has been connected such that $V_C = I\ (\Delta R)$ in series with ($R + \Delta R$) when all other sources in the network are replaced by their internal resistances.

In Compensation Theorem, the source voltage (VC) opposes the original current. In simple words compensation theorem can be stated as – the resistance of any network can be replaced by a voltage source, having the same voltage as the voltage drop across the resistance which is replaced.



Another approach:
Let the load resistance RL be changed to (RL + ΔRL). Since the rest of the circuit remains unchanged, the Thevenin's equivalent network remains the same as shown in the circuit diagram below
Here,

$$I = \frac{V_0}{R_{TH} + R_L} \ldots\ldots\ldots\ldots(1)$$



Thevenin's equivalent source

Source network with source replaced by internal resistance

$$I' = \frac{V_0}{R_{TH} + (R_L + \Delta R_L)} \ldots\ldots\ldots\ldots(2)$$

The change of current being termed as ΔI
Therefore,

$$\Delta I = I' - I \ldots\ldots\ldots\ldots\ldots(3)$$

Putting the value of I' and I from the equation (1) and (2) in the equation (3) we will get the following equation

$$\Delta I = \frac{V_0}{R_{TH} + (R_L + \Delta R_L)} - \frac{V_0}{R_{TH} + R_L}$$

$$\Delta I = \frac{V_0\{R_{TH} + R_L - (R_{TH} + R_L + \Delta R_L)\}}{(R_{TH} + R_L + \Delta R_L)(R_{TH} + R_L)}$$

$$\Delta I = -\left[\frac{V_0}{R_{TH} + R_L}\right]\frac{\Delta R_L}{R_{TH} + R_L + \Delta R_L} \ldots\ldots\ldots\ldots(4)$$

Now, putting the value of I from the equation (1) in the equation (4), we will get the following equation

$$\Delta I = -\frac{I\,\Delta R_L}{R_{TH} + R_L + \Delta R_L} \ldots\ldots\ldots\ldots(5)$$

As we know, VC = I Δ RL and is known as compensating voltage.
Therefore, the equation (5) becomes

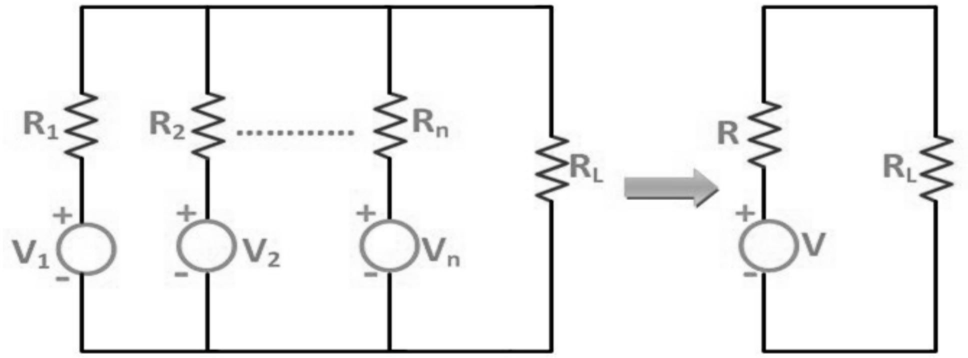
$$\Delta I = \frac{-V_C}{R_{TH} + R_L + \Delta R_L}$$

Hence, Compensation Theorem tells that with the change of branch resistance, branch currents changes and the change is equivalent to an ideal compensating voltage source in series with the branch opposing the original current, all other sources in the network being replaced by their internal resistances.

Millman's theorem

The **Millman's Theorem** states that – when a number of voltage sources (V1, V2, V3……… Vn) are in parallel having internal resistance (R1, R2, R3…………Rn) respectively, the arrangement can replace by a single equivalent voltage source V in series with an equivalent series resistance R. In other words; it determines the voltage across the parallel branches of the circuit, which have more than one voltage sources, i.e., reduces the complexity of the electrical circuit.

This Theorem is given by Jacob Millman. The utility of **Millman's Theorem** is that the number of parallel voltage sources can be reduced to one equivalent source. It is applicable only to solve the parallel branch with one resistance connected to one voltage source or current source. It is also used in solving network having an unbalanced bridge circuit.



As per Millman's Theorem

$$V = \frac{\pm V_1 G_1 \pm V_2 G_2 \pm \dots \dots \dots \pm V_n G_n}{G_1 + G_2 + \dots \dots . + G_n} \quad \text{and}$$

$$R = \frac{1}{G} = \frac{1}{G_1 + G_2 + ,,,,,,, + G_n}$$

Tellegen's Theorem

Tellegen's Theorem states that the summation of power delivered is zero for each branch of any electrical network at any instant of time. It is mainly applicable for designing the filters in signal processings. It is also used in complex operation systems for regulating the stability. It is mostly used in the chemical and biological system and for finding the dynamic behaviour of the physical network. Tellegen's theorem is independent of the network elements.Thus, it is applicable for any lump system that has linear, active, passive and time-variant elements. Also, the theorem is convenient for the network which follows Kirchoff's current law and Kirchoff's voltage law.

The steps in applying the Tellegen's theorem are as follows:

Step 1 – The following steps are given below to solve any electrical network by Tellegen's Theorem
Step 2 – In order to justify this theorem in an electrical network, the first step is to find the branch voltage drops.

Step 3 – Find the corresponding branch currents using conventional analysis methods.
Step 4 – Tellegen's Theorem can then be justified by summing the products of all branch voltages and currents.

For example, if a network having some branches "b" then

$$\sum_{b=1}^{b} v_b i_b = 0$$

Now if the set of voltages and currents is taken, corresponding the two different instants of time, t1 and t2, the Tellegen's Theorem is also applicable where we get the equation as shown below

$$\sum_{b=1}^{b} v_b\,(t_1)\,i_b\,(t_2) = \sum_{b=1}^{b} v_b\,(t_2)\,i_b\,(t_1) = 0$$

Application of Tellegen's Theorem

The various applications of the Tellegen's theorem are as follows:

- It is used in the digital signal processing system for designing of filters.
- In the area of the biological and chemical process.
- In topology and structure of reaction network analysis.
- The theorem is used in chemical plants and oil industries to determine the stability of any complex systems.

Star delta transformations

Each resistor in delta is equal to sum of all possible products of star resistors taken two at a time, divided by the opposite star resistor.

Delta-Star conversion:

R_2 is opposite to R_A; therefore,

$$R_A = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_2}$$

Also, R_1 is opposite to R_B, so

$$R_B = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1}$$

and R_3 is opposite to R_C, so

$$R_C = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_3}$$

Star-Delta conversion:

$$R_1 = \frac{R_AR_C}{R_A + R_B + R_C}$$

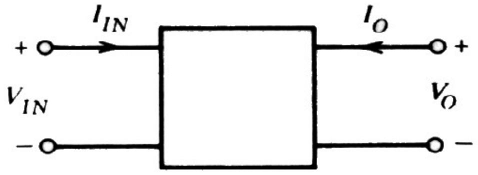
$$R_2 = \frac{R_BR_C}{R_A + R_B + R_C}$$

$$R_3 = \frac{R_AR_B}{R_A + R_B + R_C}$$

2.6 Network Functions

A network function is the Laplace transform of an impulse response. Its format is a ratio of two polynomials of the complex frequencies. Consider the general two-port network shown in Figure (a). The terminal voltages and currents of the two-port can be related by two classes of network functions, namely, the driving point functions and the transfer functions.

Figure (a)



Two port Network

I_{IN} =input current, V_{IN}=input voltage, V_0=output voltage,I_0= output current

The driving point functions relate the voltage at a port to the current at the same port. Thus, these functions are a property of a single port. For the input port the driving point impedance function $Z_{IN}(s)$ is defined as:

$$Z_{IN}(s) = \frac{V_{IN}(s)}{I_{IN}(s)}$$

[Note: f(s) denoted Laplace transform of f(t)]

This function can be measured by observing the current I_{IN} when the input port is driven by a voltage source V_{IN} (Figure (a)). The driving point admittance function $Y_{IN}(s)$ is the reciprocal of the impedance function, and is given by:

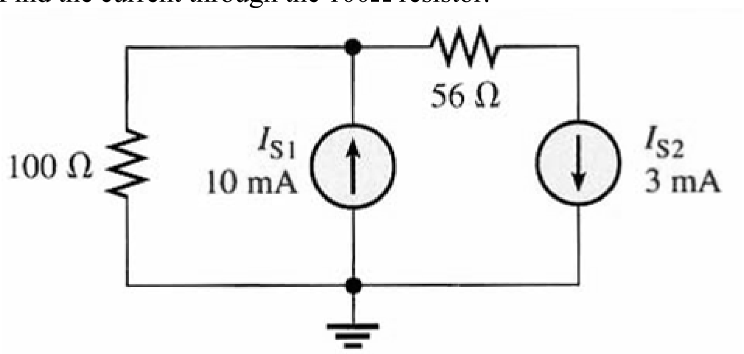
$$Y_{IN}(s) = \frac{I_{IN}(s)}{V_{IN}(s)}$$

The output port driving point functions are denned in a similar way. The transfer functions of the two-port relate the voltage (or current) at one port to the voltage (or current) at the other port. The possible forms of transfer functions are:

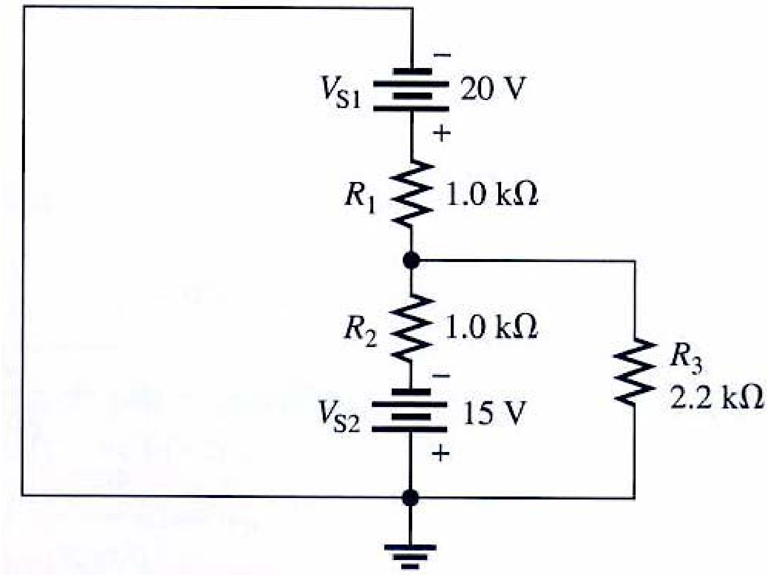
1. The voltage transfer function, which is a ratio of one voltage to another voltage.
2. The current transfer function, which is a ratio of one current to another current.
3. The transfer impedance function, which is the ratio of a voltage to a current.
4. The transfer admittance function, which is the ratio of a current to a voltage.

2.7 Assignments

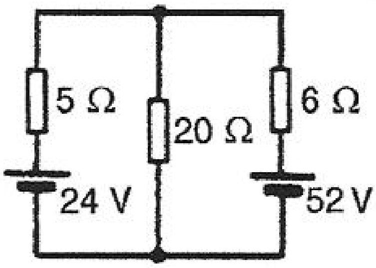
1. Find the current through the 100Ω resistor.



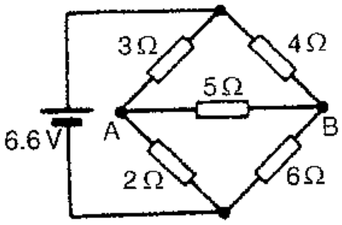
2. Find the total current through R_3.



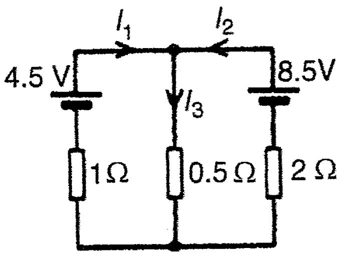
3. Use Thevenin's theorem to determine the current in each branch of the arrangement shown.



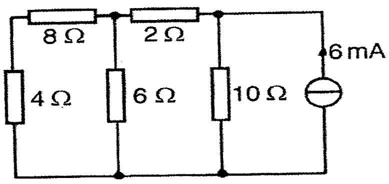
4. For the bridge network shown below, find the current in the 5 Ω resistor, and its direction, by using Thevenin's theorem.



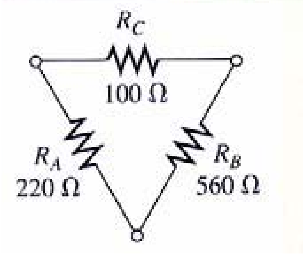
5. Use Norton's theorem to find currents I_1, I_2 and I_3 of the circuit shown.



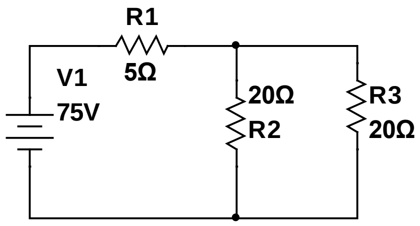
6. Determine the current flowing in the 6 Ω resistance of the network shown below by using Norton's theorem.



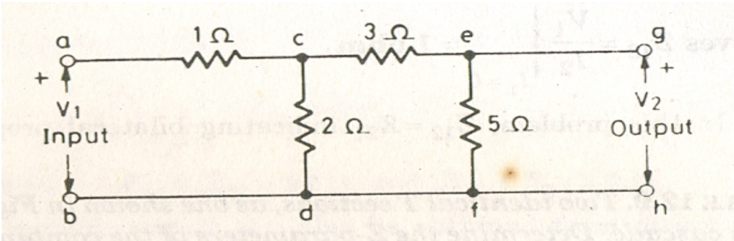
7. Convert the delta circuit to a star circuit.



8. Calculate the values of new currents in the network illustrated below when the resistor R_3 is increases by 30%.



9. Find the driving point Impedance (Z) at 'a-b' port, and transfer Admittance (Y) parameters for the circuit show in below.



GRAPH THEORY

Graph of a network is a simplest form of electric circuit representation. All the elements (resistor, inductor, capacitor)of an electric circuit can be replaced by lines with dots at both ends to form a graph for a given network.

Basic Terminologies associated with Graph:

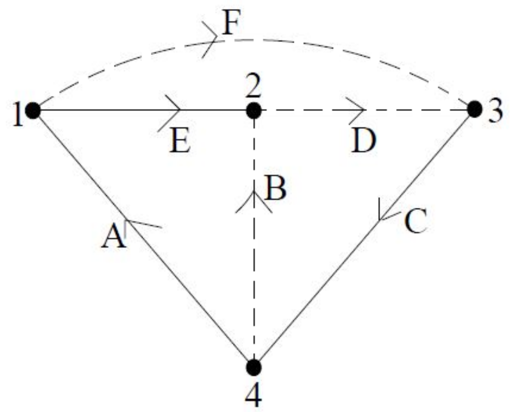


Fig 1

Branch: A branch is a line segment representing one network element/ combination of network elements connected between two points. Fig 1 shows A,B,C,D,E,F are branches.

Node: It is defined as an end point of a line segment that exists at the junction between two branches. Fig. 1 shows 1,2,3,4 are nodes.

Degree of a Node: It is the number of branches incident to it. Fig 1 shows degree of node 2 is 3.

Tree: It is an interconnected open set of branches of a graph which includes all the nodes and should never contain any loop within it. A tree is formed from a graph by deleting some branches of a graph. Fig 1 shows tree is formed by bold lines.

Twig: It is defined as the branches of a tree. Fig 1 shows A,C,E are twigs.

Tree link/ Chord: It is those branches of a graph that does not belong to that particular tree. Fig 1 shows B,D,F are links.

Directed or Oriented Graph: A graph is said to be directed when all the branches and nodes are numbered and branches are assigned with directions (arrows).

Relation between Twigs & Links

Let,

N = No of nodes; B = No of branches of a graph;L = No. of links, T = No. of twigs,
Then,
T = N-1 ; B = T+L
So, L = B – N + 1

Properties of a Tree

i. Consists of all the nodes of the graph.
ii. If a tree is obtained from a graph having N branches, then number of twigs will be N-1.
iii. A tree should not have any closed loop.
iv. For a particular graph, different trees can be obtained.

Formation of graph from electric circuit

The basic electrical circuit elements should be replaced as follows:

Resistor – By a shorted line between two nodes.

Inductor - By a shorted line between two nodes.

Capacitor - By a shorted line between two nodes.

Voltage source - By a shorted line between two nodes.

Current Source - By an open circuit.

Illustration:

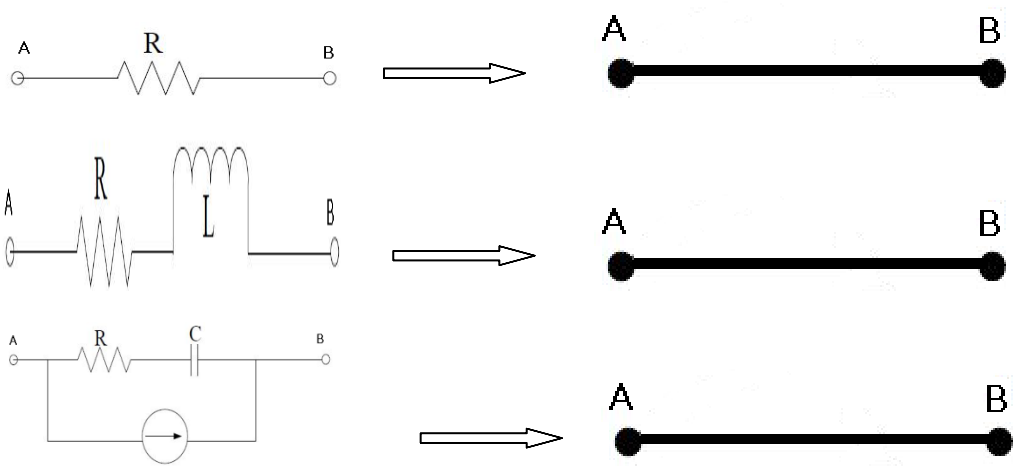


Fig. 2

Incidence Matrix

This matrix gives relation between nodes & branches of a graph. It clearly indicates whether a branch is leaving a node or entering the node.

Formation of Incidence Matrix:

Let, N = Number of nodes of graph ; B = Number of branches of graph; I_{ij} = Incidence matrix of N x B dimension;

i. If branch j is oriented away from node i, the matrix element $I_{ij} = 1$.
ii. If branch j is oriented towards node i, the matrix element $I_{ij} = -1$.
iii. If branch j is not incident at node I, the matrix element $I_{ij} = 0$.

The complete set of incidence matrix is called augmented incidence matrix.

Note that algebraic sum of the column entries of an incidence matrix is always zero.

Example I: Obtain incidence matrix of the graph shown in figure below:

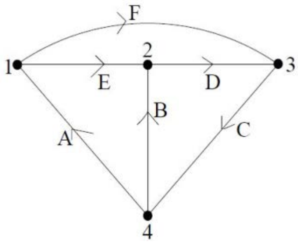


Fig. 3

Node ↓ Branch ⟶ A B C D E F

$$Iij = \begin{bmatrix} -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & -1 \\ 1 & 1 & -1 & 0 & 0 & 0 \end{bmatrix}$$

It is observed that column wise algebraic sum is zero. This is because a branch can leave a particular node and enters a particular node. It is a N x B matrix.

Reduced incidence Matrix:

It is possible to delete any one row from incidence matrix I_{ij} without loosing any information about graph. The matrix obtained from incidence matrix by deleting one row is known as Reduced incidence matrix.

For a graph having N nodes & B branches, the order of the reduced incidence matrix is (N – 1) x B.

Example II: Obtain reduced incidence matrix of the graph shown in figure below:

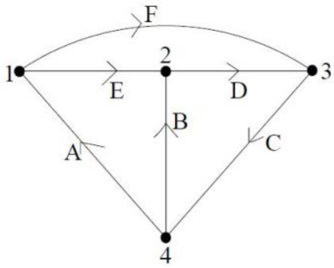


Fig. 4

Node ↓ Branch ⟶ A B C D E F

$$Iij = \begin{bmatrix} -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & -1 \end{bmatrix}$$

Here node 4 (4^{th} row) is deleted to obtain reduced incidence matrix. Note that as column wise algebraic sum is zero, hence it can easily be understood that what should be 4^{th} row matrix elements.

Fundamental Tie Set Matrix or Fundamental Loop Matrix:

A fundamental Tie set is formed by choosing a tree from a given graph. The fundamental loops should have only one link and other twigs. The number of fundamental loops is equal to number of links.

Formation of Fundamental Tie Set Matrix:

Step I: Choose a tree arbitrarily.

Step II: Form fundamental loops with one link & other twigs.

Step III: Assume direction of loop currents in the direction of links.

Step IV: Form fundamental Tie set Matrix T_{ij} as follows;

i. If branch j is oriented in the direction of loop i, $T_{ij} = 1$.
ii. If branch j is oriented in opposite direction of loop i, $T_{ij} = -1$.
iii. If branch j is not part of loop i, $T_{ij} = 0$.

Example III: Draw graph for the circuit given in fig 5 and obtain Tie set matrix.

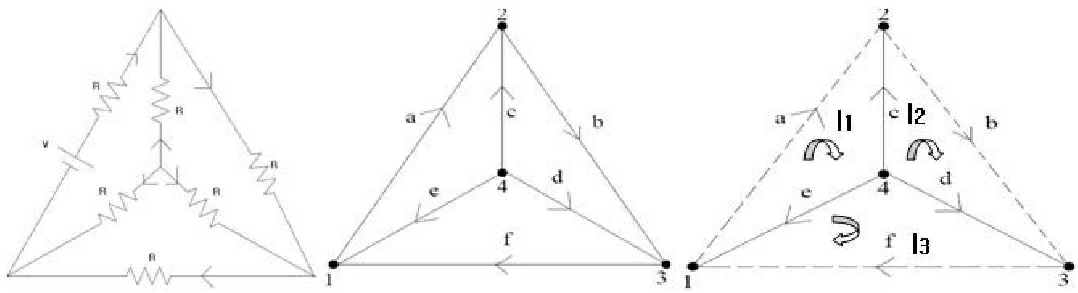


Fig. 5

Here, dotted lines show links and bold lines show twigs. The loop currents are assumed in the direction of links.

Loop currents **Branch** ⟶ **a b c d e f**

↓

$$Tij = \begin{bmatrix} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{bmatrix}$$

Fundamental Cut Set Matrix:

A fundamental cut set of a graph with respect to a tree is formed by one twig & other links. The number of cut sets of a graph with respect to a tree is equal to number of twigs.

Formation of Fundamental Cut Set Matrix:

Step I: Choose a tree arbitrarily.

Step II: Form fundamental cut sets with one twig & other links.

Step III: Assume direction of cut sets in the direction of twigs.

Step IV: Form fundamental Cut set Matrix C_{ij} as follows;

i. If branch j has same orientation of cut set i, $C_{ij} = 1$.
ii. If branch j has opposite orientation of cut set i, $C_{ij} = -1$.
iii. If branch j is not part of cut set i, $C_{ij} = 0$.

Example IV: Obtain cut set matrix of the graph shown in figure 6:

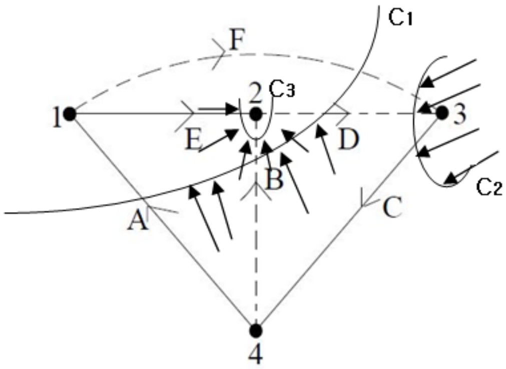


Fig. 6

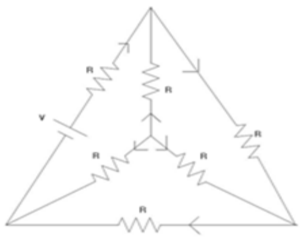
Cut Set C_1 is formed with Twig A, Links B &D.
Cut Set C_2 is formed with Twig C, Links B &D.
Cut Set C_3 is formed with Twig E, Links F &D.

Cut set ↓ **Branch** ⟶ **A B C D E F**

$$Iij = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{bmatrix}$$

Exercise:

1. Discuss significance of
   (a) Incidence matrix
   (b) Tie set matrix
   (c) Cut set matrix
   (d) Reduced incidence matrix in electrical circuit analysis.
2. For the following electrical circuit, obtain (a) Graph (b) All possible combination of tree (c) Incidence matrix (d) cut set matrix



COUPLED CIRCUITS

Magnetic flux:

Magnetic flux (Φ_B) is the number of magnetic field lines passing through a surface (such as a loop of wire). The magnetic flux through a closed surface (such as a ball) is always zero. This implies that there cannot be magnetic charges in classical electromagnetic.

The SI unit of magnetic flux is the Weber (Wb) (in derived units: volt-seconds). The CGS unit is the Maxwell.

Magnetic flux is sometimes used by electrical engineers designing systems with electromagnets or designing dynamos. Physicists designing particle accelerators also calculate magnetic flux.

$\Phi = \iint_S B.dA$

where

Φ is the magnetic flux
B is the magnetic field
S is the surface area
denotes dot product
dA is the infinitesimal vector

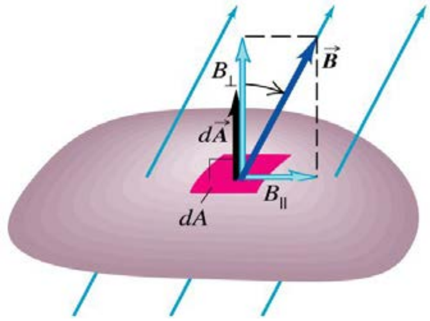


Fig 1: Magnetic Flux

Self-Inductance:

Self-inductance or in other words inductance of the coil is defined as the property of the coil due to which it opposes the change of current flowing through it. Inductance is attained by a coil due to the self-induced emf produced in the coil itself by changing the current flowing through it.
If the current in the coil is increasing, the self-induced emf produced in the coil will oppose the rise of current, that means the direction of the induced emf is opposite to the applied voltage.
You can determine the self-inductance of a coil by the following expression:

$E = L\frac{di}{dt}$ or $N\frac{d\phi}{dt}$

Where,
L= inductance
I=Current flowing through coil
ϕ= Magnetic flux
N= No of turns of coil

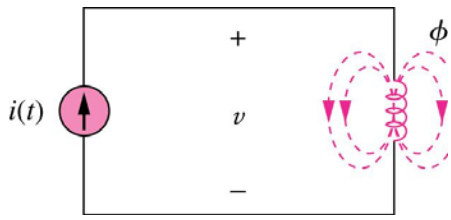


Fig 2: Self Inductance

Mutual Inductance

- Production of magnetic flux by a current
- The production of voltage by time varying magnetic field
- Current flowing in one coil established flux about that coil and about second coil nearby
- The time varying flux surrounding second coil produces a voltage across the terminals of the second coil
- This voltage is proportional to time rate of change of current in first coil
- Define coefficient of mutual inductance or simply mutual inductance

$$v_1 = N_1\frac{d\phi_1}{dt} = N_1\frac{d(\phi_{11}+\phi_{21})}{dt}$$

$$v_2 = N_2\frac{d\phi_2}{dt} = N_2\frac{d(\phi_{12}+\phi_{22})}{dt}$$

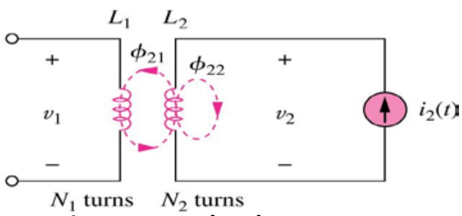


Fig 3: Mutual Inductance

$$v_2 = M_{21}\frac{di_1}{dt}$$ $$v_1 = M_{12}\frac{di_2}{dt}$$

$$M_{12} = M_{21} = M$$

Dot Convention:

- A current entering the dotted terminal of one coil produces an open circuit voltage with positive voltage reference at the dotted terminal of the second coil
- A current entering the un dotted terminal of one coil provides a voltage that is positively sensed at the un dotted terminal of the second coil

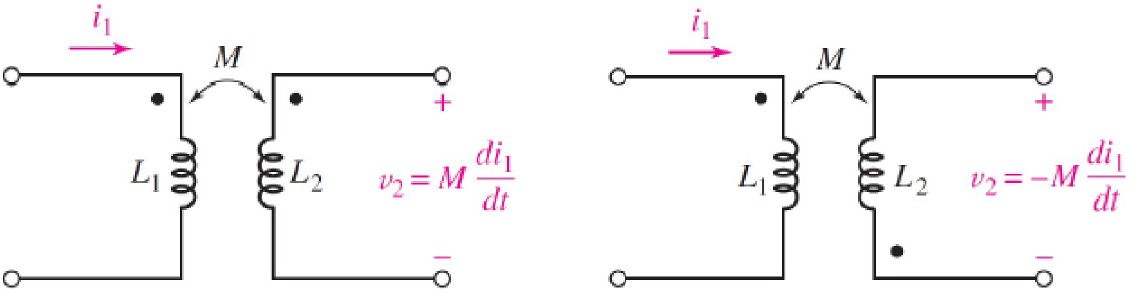


Fig 4:

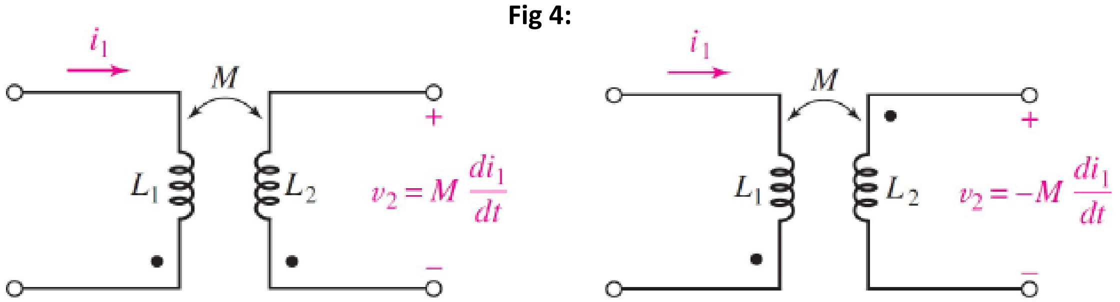


Fig 5:

Coils in series:

The total inductance of two coupled coils in series depend on the placement of the dotted ends of the coils. The mutual inductances may add or subtract.

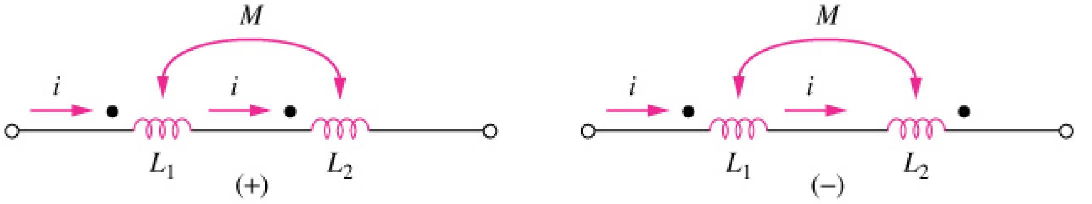


Fig 6:

(a) Series aiding Connection

$L=L_1+L_2+2M$

(b) Series opposing Connection

$L=L_1+L_2-2M$

Example: (a) determine v1 if i2 = 5 sin 45t A and i1 = 0;

(b) Determine v2 if i1=−8e−t A and i2 = 0 di_2

The current i2 (entering undotted terminal) results in a positive reference for the voltage induced across the left coil is the undotted terminal

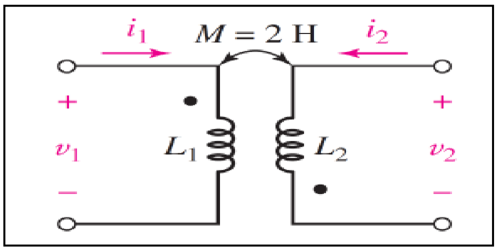


Fig 7:

Ans: $V_1(t)=\frac{di2t}{dt}=-450\cos 45t$

$V_2(t)=\frac{di1t}{dt}= -16\ e^{-t}\ V$

Time Domain and Frequency Domain Analysis:

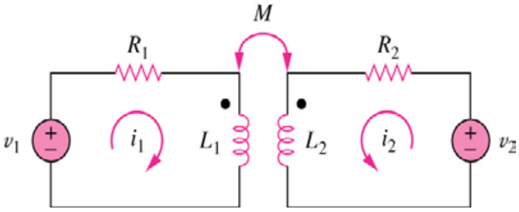


Fig 8: Time Domain Circuit

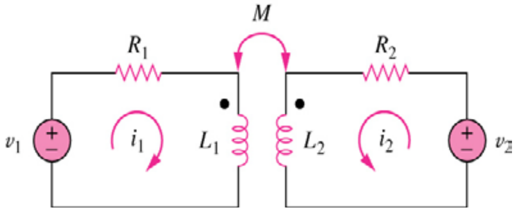


Fig 9: Frequency Domain Circuit

Time Domain

$$v_1 = i_1 R_1 + L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2 = i_2 R_2 + L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

Frequency Domain

$$V_1 = (R_1 + j\omega L_1) I_1 + j\omega M I_2$$

$$V_2 = j\omega M I_1 + (R_2 + j\omega L_2) I$$

Energy in Coupled Circuit:

The total energy *w* stored in a mutually coupled inductor is:

- **Positive sign** is selected if both currents ENTER or LEAVE the dotted terminals.
- Otherwise we use Negative sign.

$$w = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 \pm M i_1 i_2$$

Coupling Coefficient:

The Coupling Coefficient *k* is a measure of the magnetic coupling between two coils $0 \le k \le 1$

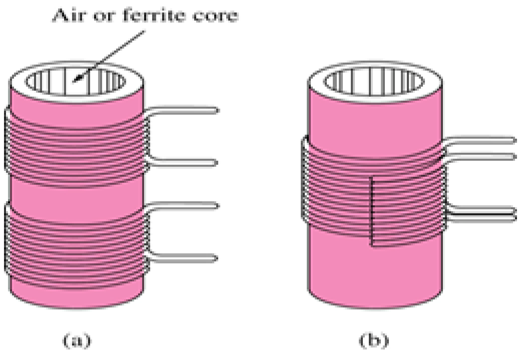
$$0 \le k \le 1$$

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

K = 1 perfect coupling

K < 0.5 Loosely Coupling

K > 0.5 Tightly coupling

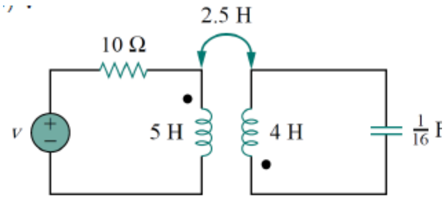


(a) Loosely Coupled Circuit (b) Tightly Coupled

Fig 10:

Example 1:

Determine the coupling coefficient. Calculate the stored energy in the coupled inductors at t=1 s if v= 60 cos(4t+30) V.



Ans:

The coupling coefficient is

$$K=\frac{M}{\sqrt{L1*L2}}=\frac{2.5}{\sqrt{20}}=0.56$$

After making mesh analysis we can found out that,

I_2=3. 254 $\angle 160.60^0$ A

I_1= -1.2 I_2

In the time domain

I_1= 3.095cos (4t- 19.4^0)

I_2= 3.254 cos (4t- 199.4^0)

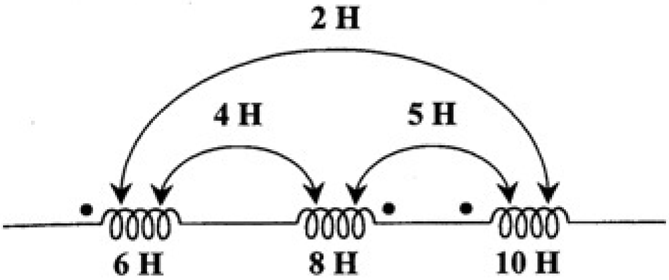
The total energy stored in the coupled inductors is

$$W=\frac{1}{2}L_1I_1^2+\frac{1}{2}L_2I_2^2+MI_1I_2$$

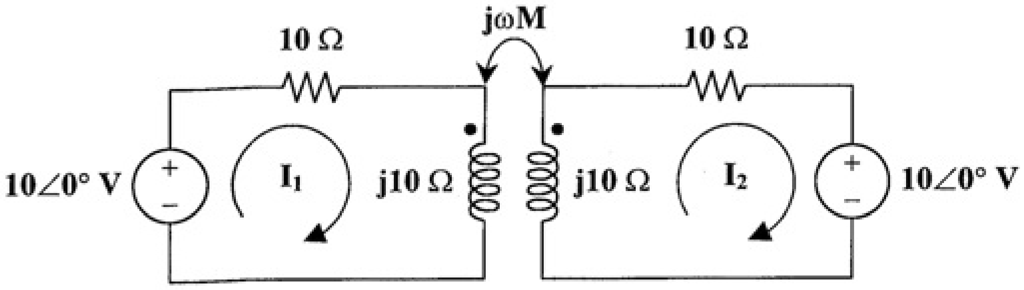
$$=20.73\text{ J}$$

Exercise:

1. Determine equivalent inductance of the circuit given below



2. For k = 1 find I_1 & I_2.



LAPLACE TRANSFORM

The **Laplace Transform** is a powerful tool that is very useful in Electrical Engineering. The transform allows equations in the "time domain" to be transformed into an equivalent equation in the **Complex S Domain**.

The mathematical definition of the Laplace transform is as follows:

$$F(s) = \mathcal{L}\{f(t)\} = \int_{0^-}^{\infty} e^{-st} f(t)\, dt$$

Laplace Domain

The **Laplace domain**, or the "Complex s Domain" is the domain into which the Laplace transform transforms a time-domain equation. s is a complex variable, composed of real and imaginary parts:

$$s = \sigma + j\omega$$

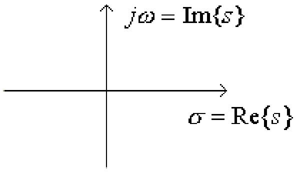


Fig 3.1 S-plane

The Laplace domain graphs the real part (σ) as the horizontal axis, and the imaginary part (ω) as the vertical axis. The real and imaginary parts of s can be considered as independent quantities.

Properties of Laplace Transformer:

Laplace transform of derivative-

$$\mathcal{L}\{f'(t)\} = sF(s) - f(0)$$

Likewise, we can express higher-order derivatives in a similar manner:

$$\mathcal{L}\{f''(t)\} = s^2F(s) - sf(0) - f'(0)$$

For integrals, we get the following:

$$\mathcal{L}\left\{\int_0^t f(t)\, dt\right\} = \frac{1}{s}F(s)$$

Initial Value Theorem

The **Initial Value Theorem** of the laplace transform states as follows:

$$f(0) \Leftrightarrow \lim_{s\to\infty} sF(s)$$

N.B-This is useful for finding the initial conditions of a function needed when we perform the transform of a differentiation operation

Final Value Theorem

Similar to the Initial Value Theorem, the **Final Value Theorem** states that we can find the value of a function f, as t approaches infinity, in the laplace domain, as such

$$\lim_{t\to\infty} f(t) \Leftrightarrow \lim_{s\to 0} sF(s)$$

N.B-This is useful for finding the steady state response of a circuit. The final value theorem may only be applied to stable systems.

Laplace transform of different function

$f(t)\quad (t \geq 0)$	$F(s)$
$\delta(t)$ (unit impulse)	1
$u(t)$ (unit step)	$\frac{1}{s}$
t (unit ramp)	$\frac{1}{s^2}$
t^n (n>-1)	$\frac{n!}{s^{n+1}}$
e^{-at}	$\frac{1}{s+a}$
$\sin bt$	$\frac{b}{s^2+b^2}$
$\cos bt$	$\frac{s}{s^2+b^2}$
$e^{-at}\sin\omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos\omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

Transfer Function-

A transfer function G(s)or H(s) is defined as the ratio of Laplace transform of the output C(s) to Laplace transform of input R(s)

$$G(S) = \frac{C(S)}{R(S)}$$

Poles & Zeros :

The roots of C(s)(numerator) for which the value of transfer function become zero is known as zeros.

The roots of R(s) (denominator) for which G(s) become infinite is known as poles.

Example

Find the pole-zero representation of the system with the transfer function:

$$H(s) = \frac{6s^2 + 18s + 12}{2s^3 + 10s^2 + 16s + 12}$$

First rewrite in our standard form (note: the polynomials were factored with a computer).

$$H(s) = \frac{6}{2}\frac{s^2 + 3s + 2}{s^3 + 5s^2 + 8s + 6} = 3\frac{(s+1)(s+2)}{(s+1-j)(s+1+j)(s+3)} \qquad j = \sqrt{-1}$$

So the pole-zero representation consists of:

- zeros at s=-1 and s=-2, and
- polese at s=-1+j, s=-1-j and s=-3.

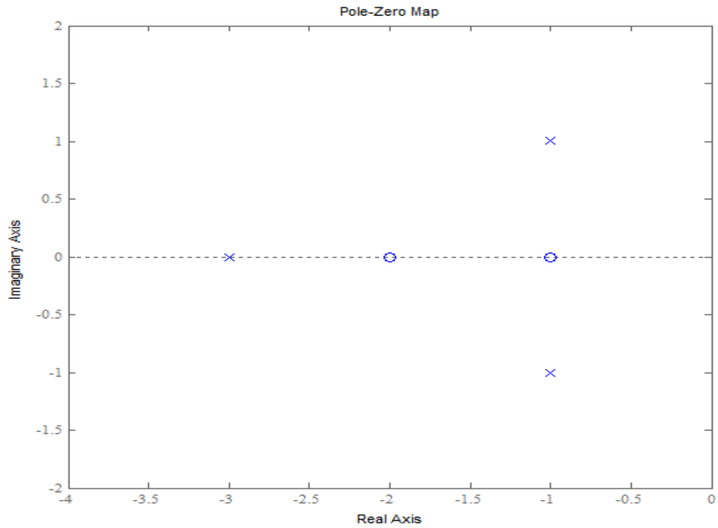


Fig 3: Pole Zero Mapping

Properties of Lapalce transform:

Property 1. Constant Multiple

If a is a constant and $f(t)$ is a function of t, then

$$\mathcal{L}\{a\cdot f(t)\} = a\cdot\mathcal{L}\{f(t)\}$$

Property 2. Linearity Property

If a and b are constants while $f(t)$ and $g(t)$ are functions of t, then

$$\mathcal{L}\{a\cdot f(t) + b\cdot g(t)\} = a\cdot\mathcal{L}\{f(t)\} + b\cdot\mathcal{L}\{g(t)\}$$

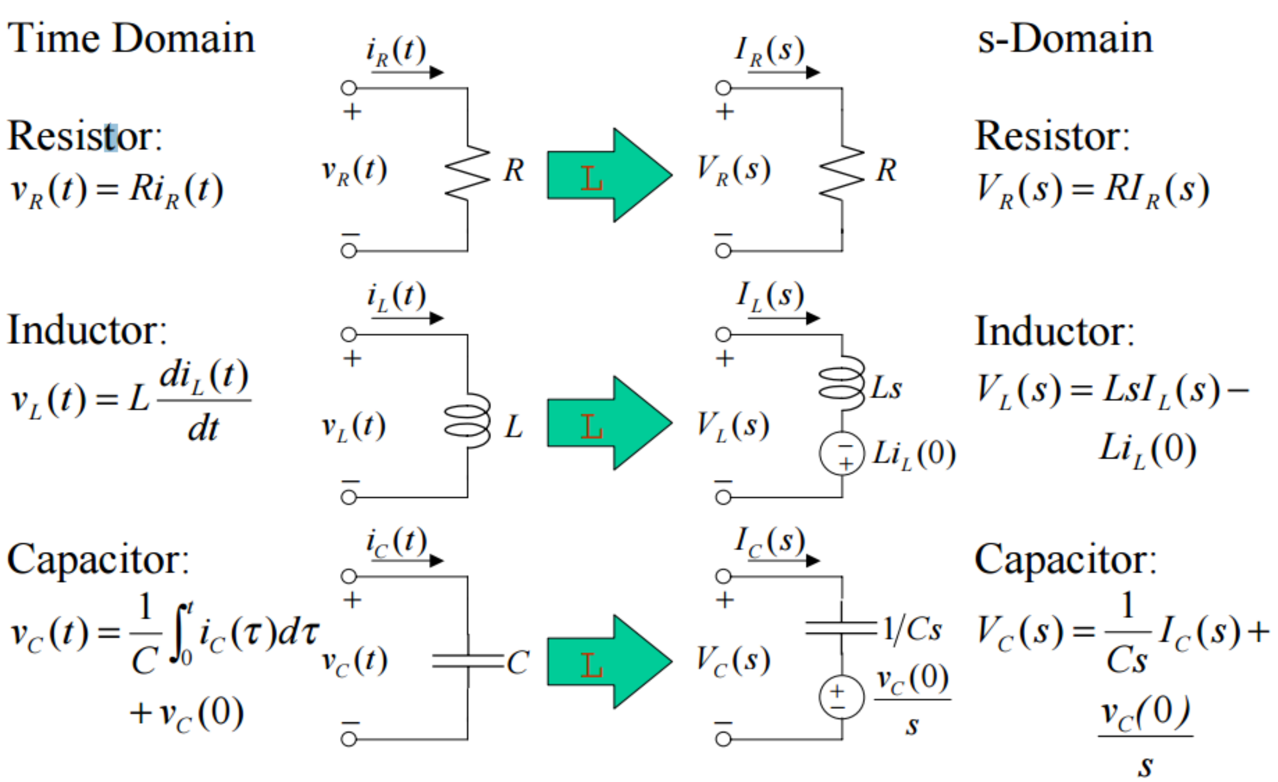
Property 3. Change of Scale Property

If $\mathcal{L}\{f(t)\} = F(s)$ then $\mathcal{L}\{f(at)\} = \frac{1}{a}F\left(\frac{s}{a}\right)$

Property 4. Shifting Property (Shift Theorem)

$$\mathcal{L}\{e^{at}f(t)\} = F(s-a)$$

Circuit Elements in S domain



Exercise:

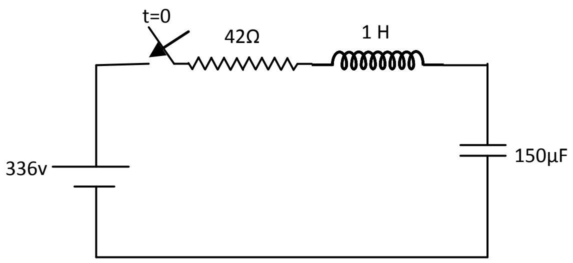
Problem 1:
Find the Laplace transform of the ramp function r(t) = t

Problem 2:
Find the transform of the function

$F(t) = (2 - 2e^{-2t}) / t$

Problem 3:
Derive an expression for the Laplace transform of the derivative of a function.

Problem 4:
Using Laplace transform obtain the charging current expression.



Problem 5:
State and Prove the Convolution Theorem.

Objectives:
1. Laplace transform analysis gives
a) time domain response only b) frequency domain response only c) both a &b d) none

2.The laplace transform of first derivative of a function f(t) is
a) F(s)/s b) sF(s)-f(0) c) F(s)-f(0) d)f(0)

3. The laplace transform of integral of a function f(t) is
a) F(s)/s b) sF(s)-f(0) c) F(s)-f(0) d)f(0)

4. A 10 Ω resistor, a 1 H inductor and 1 μF capacitor are connected in parallel. The combination is driven by a unit step current. Under the steady state condition, the source current flows through.
a) the resistor b) the inductor c) the capacitor only d) all the three elements

5. The Steady state value can be calculated using ....
a) initial value b)final value theorem c)total response d)natural response

6. Sudden change in current is not occurred in which component
a) capacitor b)inductor c)resistor d)natural response

Module VI

CIRCUIT TRANSIENTS

What is transient?

The time varying currents and voltages resulting from the sudden application of source, usally due to switching, are called trnsients. By writing circuit equations, we obtain integral differential equations. Transient & Stedy State Response

Complete response = Transient Response + Steady State Response

Transient Response ↓ Temporary Part

Steady State Response ↓ Permanent Part

System Response

In Equilibrium (Steady State Response)

Change in Equilibrium (Transient Response)

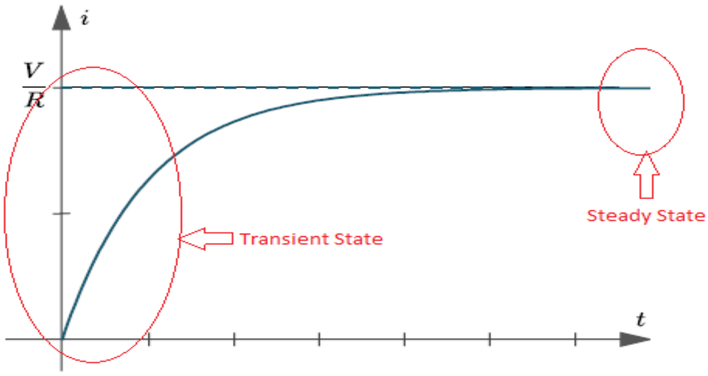
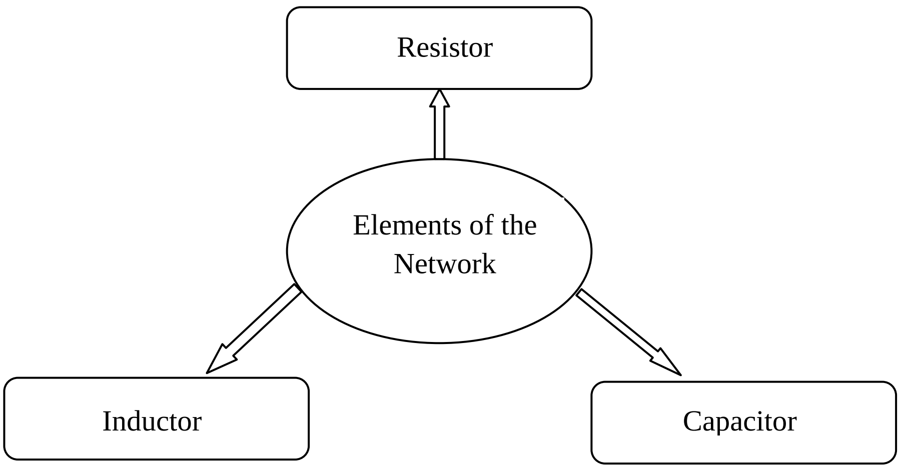


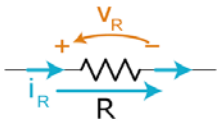
Fig1. Transient & Steady State Response

Transient response is the response of a system from rest or equilibrium to steady state.
The values of voltage and current after the transient has died out are known as the steady state responses.

Basic Elements of the Network:

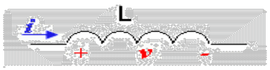


Resistor: A resistor is a two terminal electronic component. It produces a voltage across its terminals that is proportional to the current following through it in accordance with Ohm's Law:
V = IR



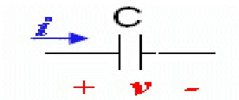
Thus, in resistor, change in current is instantaneous as there is no storage of energy in it, in units of Ohm.

Inductor: An inductor is a passive component that can store energy in a magnetic field created by the current passing through it. An inductor's ability to store magnetic energy is measure by its inductance, in units of Henries. Current in an inductor cannot change instantaneously.

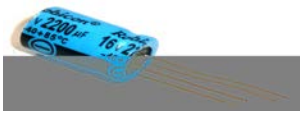


$$v(t) = L\frac{d}{dt}i(t) \qquad i(t) = \frac{1}{L}\int_{t_0}^{t} v(\tau)dt + i(t_0)$$

Capacitor: The current through a capacitor can be changed instantly, but it takes time to change the voltage across a capacitor. Units is Farads.



$$i(t) = C\frac{d}{dt}v(t) \qquad v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)dt + v(t_0)$$



Passive Linear Circuit Elements

	Resistor	Capacitor	Inductor
Current Voltage Characteristic	V = IR	$i = C\frac{dv}{dt}$	$v = L\frac{di}{dt}$
Series Equivalent	$R_{eq} = \frac{R_1R_2}{R_1+R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1L_2}{L_1+L_2}$
Parallel Equivalent	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1C_2}{C_1+C_2}$	$L_{eq} = L_1 + L_2$

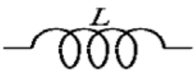
Complex Impedance	$z(s) = \dfrac{V(s)}{I(s)}$	$z(s) = \dfrac{1}{sC} = \dfrac{1}{j\omega c}$	$z(s) = sL = j\omega L$
Time Constant (With Resistor)		$\tau = RC$	$\tau = \dfrac{L}{R}$

Initial Conditions: We need initial conditions to evaluate the arbitrary constants in the general solution of differential equations. We find the change in selected variables in a circuit when one or more switches are moved from open to closed positions or vice versa. The representation of initial conditions of RLC circuit excited by a DC source is shown in table 1

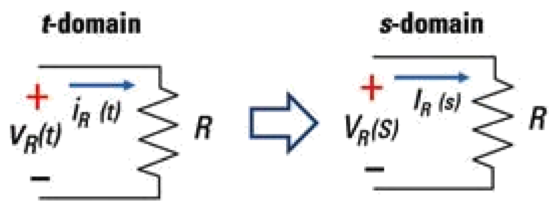
- **t = 0^-** indicated the time just before changing the position of the switch
- **t = 0** indicated the time when the position of switch is changed
- **t = 0^+** indicated the time immediately after changing the position of switch

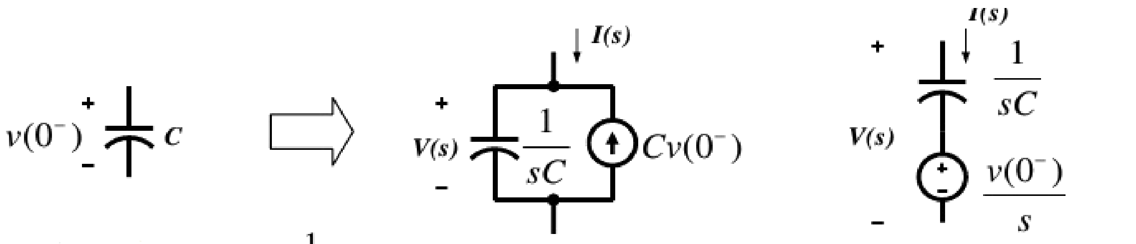
Initial condition for the resistor: For a resistor current and voltage are related by v(t) = Ri(t). The current through a resistor will change instantaneously if the voltage changes instantaneously. Similarly the voltage will change instantaneously, if the current changes instantaneously.
Initial condition for the inductor: Voltage across the inductor is proportional to the rate of change of current. It is impossible to change the current through an inductor by a finite amount in zero time. This requires an infinite voltage across the inductor.
Initial condition for the capacitor: Current through a capacitor is proportional to the rate of change of voltage. Due to this change the voltage across a capacitor by a amount in zero time. This requires an finite current through the capacitor.

Table1: representation of initial and final conditions of an RLC circuit excited by a DC source

Circuit Elements	**at initial conditions t = 0+**	**at final conditions t = ∞**
		
	 If there is no current flowing through an inductor at t=0, the inductor will act as open circuit at t = 0^+	
	 If there is no voltage across the capacitor at t=0^-, the capacitor will act as a short at t = 0^+	

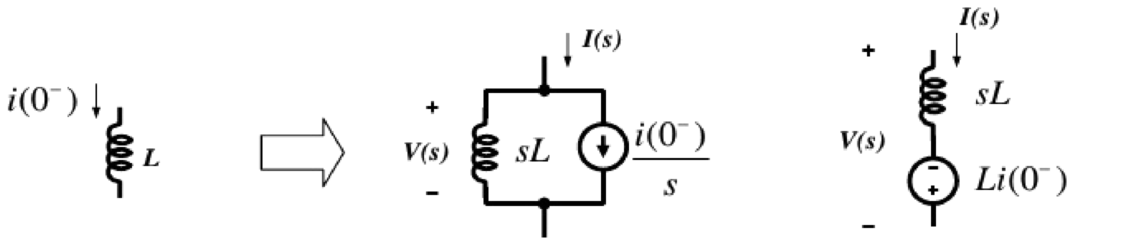
Representation of RLC devices in time-domain and s-Domain:





Capacitor has an impedance: $\frac{1}{sC}$ $\quad I(s) = sCV(s) - Cv(0^-) \quad V(s) = \frac{1}{sC}I(s) + \frac{v(0^-)}{s}$

–



Inductor has an impedance: sL $\quad I(s) = \frac{1}{sL}V(s) + \frac{i(0^-)}{s} \quad V(s) = sLI(s) - Li(0^-)$

Solution to First Order Differential Equation:

Consider the general Equation

$$\tau \frac{dx(t)}{dt} + x(t) = K_s f(t)$$

Let the initial condition be $x(t = 0) = x(0)$, then we solve the differential equation:

$$\tau \frac{dx(t)}{dt} + x(t) = K_s f(t)$$

The complete solution consists of two parts:

- the homogeneous solution (natural solution)
- the particular solution (forced solution)

The Natural Response:

Let the general Equation:

$$\tau \frac{dx(t)}{dt} + x(t) = K_s f(t)$$

Setting the excitation $f(t)$ equal to zero,

$$\tau \frac{dx_n(t)}{dt} + x_n(t) = 0 \; or \quad \frac{dx_n(t)}{dt} = -\frac{x_n(t)}{\tau}, \; \frac{dx_n(t)}{x_n(t)} = -\frac{dt}{\tau}$$

$$\int \frac{dx_n(t)}{x_n(t)} = \int -\frac{dt}{\tau}, \qquad x_n(t) = \alpha e^{-t/\tau}$$

$X_n(t)$ is called the natural response.

The Forced Response:

Let the general equation:

$$\tau \frac{dx(t)}{dt} + x(t) = K_s f(t)$$

Setting the excitation $f(t)$ equal to F, a constant for $t \geq 0$

$$\tau \frac{dx_f(t)}{dt} + x_f(t) = K_S F$$

$$x_f(t) = K_S F \quad for \; t \geq 0$$

$X_f(t)$ is called the forced response

The Complete Response:

Let the general Equation and Solve for α,

$$\tau \frac{dx(t)}{dt} + x(t) = K_s f(t)$$

$$for \; t = 0$$

$$x(t=0) = x(0) = \alpha + x(\infty)$$

$$\alpha = x(0) - x(\infty)$$

The complete response is:

- **the natural response is**
- **the forced response is**

$$x = x_n(t) + xf(t)$$

$$= \alpha e^{-t/\tau} + K_S F$$

The Complete solution is:

$$= \alpha e^{-t/\tau} + x(\infty)$$

$$x(t) = [x(0) - x(\infty)]e^{-t/\tau} + x(\infty)$$

$$[x(0) - x(\infty)]e^{-t/\tau}$$

It is called transient response

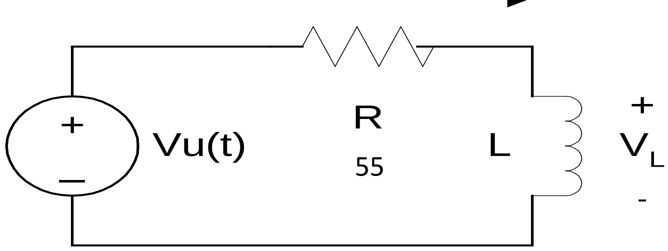
$$x(\infty)$$

It is called steady state response

Complete response = natural response + forced response

↓ **Due to the initial condition** ↓ **Due to the external excitation**

DC Response of an R-L Circuit:



$$Ri + L\frac{di}{dt} = V$$

$$\frac{Ldi}{V - Ri} = dt$$

$$-\frac{L}{R}\ln(V - Ri) = t + k$$

$$i(0^+) = 0, thus\ \ k = -\frac{L}{R}\ln V$$

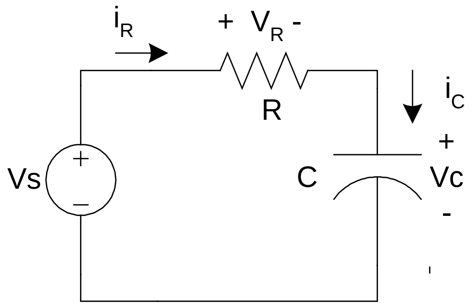
$$-\frac{L}{R}[\ln(V - Ri) - \ln V] = t$$

$$\frac{V - Ri}{V} = e^{-Rt/L} \qquad or$$

$$i = \frac{V}{R} - \frac{V}{R}e^{-Rt/L},\ \ for\, t > 0$$

Where L/R is the time constant

DC Response of an R-C Circuit:



$$i_R = i_C$$

$$i_R = \frac{v_s - v_R}{R}, i_C = C\frac{dv_C}{dt}$$

$$\frac{dv_C}{dt} + \frac{1}{RC}v_R = \frac{1}{RC}v_s$$

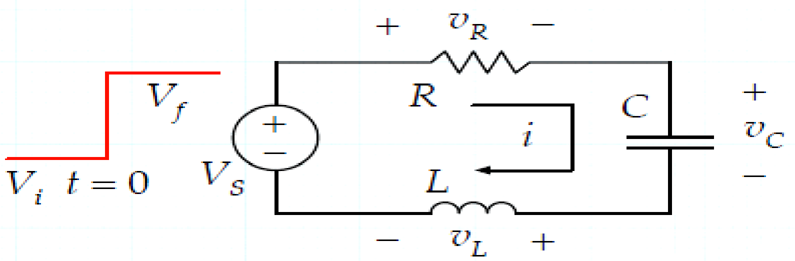
The solution will give natural response $i_R = i_C = Ke^{-t/RC}$

The corresponding voltage drops across the resistor and capacitor can be obtained as follows:

$$v_R = iR = -Ve^{-t/RC}$$

Where RC is a time constant

DC Response of an R-L-C Circuit:



$$t > 0: \quad V_f = iR + v_c + L\frac{di}{dt}$$

$$i = C\frac{dv_C}{dt}$$

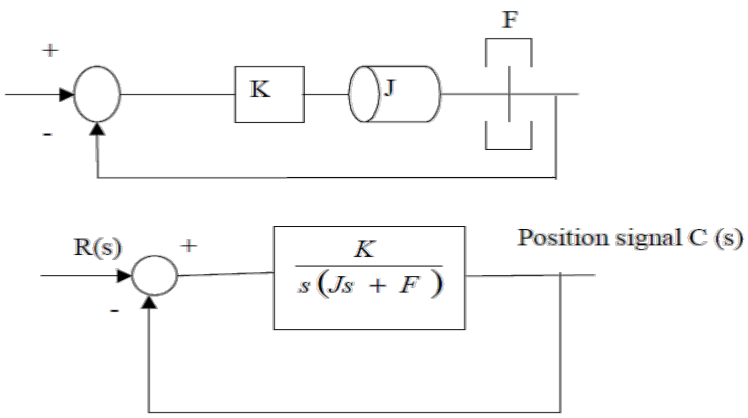
$$V_f = RC\frac{dv_C}{dt} + v_C + LC\frac{d^2 v_C}{dt^2}$$

$$\frac{d^2 v_C}{dt^2} + \frac{R}{L}\frac{dv_C}{dt} + \frac{v_C}{LC} = \frac{V_f}{LC}$$

Second-order differential equation.

2nd Order System:

Block Diagram



Transfer function:

$$\frac{C(s)}{R(s)} = \frac{K}{Js^2 + Fs + K}$$

$$= \frac{\frac{K}{J}}{\left[s + \frac{F}{2J} + \sqrt{\left(\frac{F}{2J}\right)^2 - \frac{K}{J}}\right]\left[s + \frac{F}{2J} - \sqrt{\left(\frac{F}{2J}\right)^2 - \frac{K}{J}}\right]}$$

Substitute in the transfer function:

$$\frac{K}{J} = \omega_n{}^2$$

$$\frac{F}{J} = 2\ \zeta\omega_n = 2\sigma$$

$$\zeta = \frac{F}{2\sqrt{JK}}$$

ζ: damping ratio

ω_n: undamped natural frequency

σ: stability ratio

to obtain

$$\frac{C(s)}{R(s)} = \frac{\omega_n^{\ 2}}{s^2 + 2\zeta\omega_n s + \omega_n^{\ 2}}$$

- ***Underdamped*** case: $0 < \zeta < 1$

  $F^2 - 4JK < 0$ two *complex conjugate* poles

- ***Critically damped*** case: $\zeta = 1$

  $F^2 - 4JK = 0$ two *equal real* poles

- ***Overdamped*** case: $\zeta > 1$

  $F^2 - 4JK > 0$ two *real* poles

MODULE: VII

TWO PORT NETWORK

When a number of impedances are connected together to form a system that consist of set of interconnected circuits performing specific function, is called as a network.

One Port Network: A pair of terminals at which a signal may enter or leave a network is called a port n/w or one port n/w.

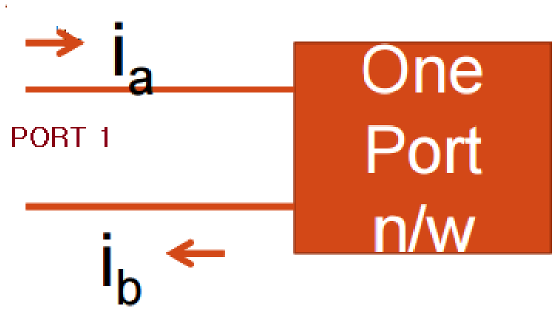


Fig 1. One port Network

Two Port Parameters It represented by a black box with four variables, two voltage (V1 , V2) and two currents (I1 , I2), which are available for measurements Out of these four variables, which two variables may be considered ‘independent’ and which two ‘dependent’ is generally decided by the problem under consideration.

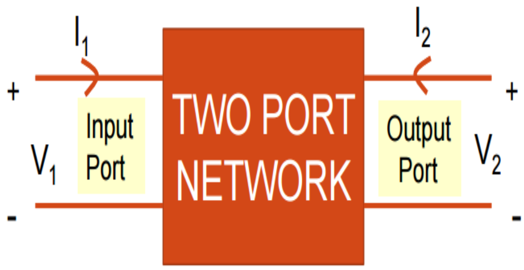


Fig 2. Two port Network

Z – PARAMETERS (Open Circuit Impedance Parameters):

The figure below shows a two port network. The voltages & currents of both ports are shown. The currents inward to the network are considered as positive.

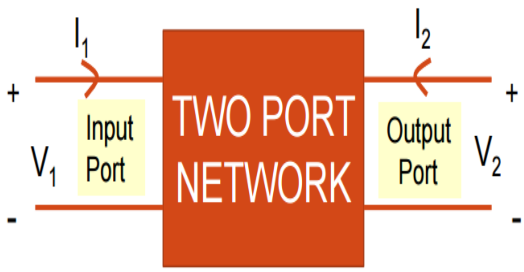


Fig 3. Two port Network

For a two port network,

$[V] = [Z][I]$……………………………………………………………………………………..(i)

$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$……………………………………………………………………… (ii)

Where, $[Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$ = Impedance Matrix

Equation (ii) can be written as,

$V_1 = Z_{11}I_1+Z_{12}I_2$ ....(iii)

$V_2 = Z_{21}I_1+Z_{22}I_2$ .... (iv)

To find

Z_{11} & Z_{21}, output port should be open circuited, i.e.; $I_2 = 0$,

So, from equation (iii), $Z_{11} = [V_1/I_1]_{I2=0}$; Known as Input driving point impedance

From equation (iv), $Z_{21} = [V_2/I_1]_{I2=0}$; Known as forward transfer impedance

To find

Z_{12} & Z_{22}, input port should be open circuited, i.e; $I_1 = 0$,

So, from equation (iii), $Z_{12} = [V_1/I_2]_{I1=0}$; Known as reverse transfer impedance

From equation (iv), $Z_{22} = [V_2/I_2]_{I1=0}$; Known as Output driving point impedance

Z_{11}, Z_{12}, Z_{21}, Z_{22} all are impedances and unit is ohm.

As Z parameters are obtained by open circuiting any of the two ports and expressed as ratio of voltage & current, these are known as **OPEN CIRCUIT IMPEDANCE PARAMETERS.**

Equivalent circuit model of Z parameter representation:

The following circuit diagram shows equivalent circuit model of Z parameters which satisfies equation (iii) & (iv).

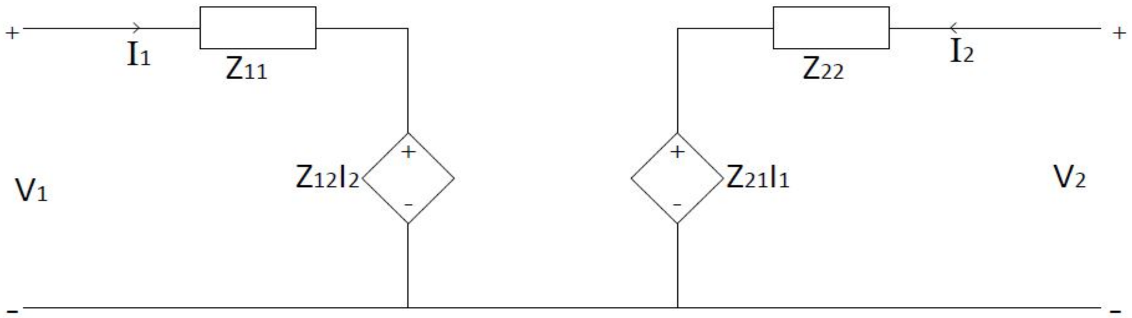


Fig 4. Equivalent circuit model of Z parameter representation

Example I: Find z parameters of the following network shown in fig 5.

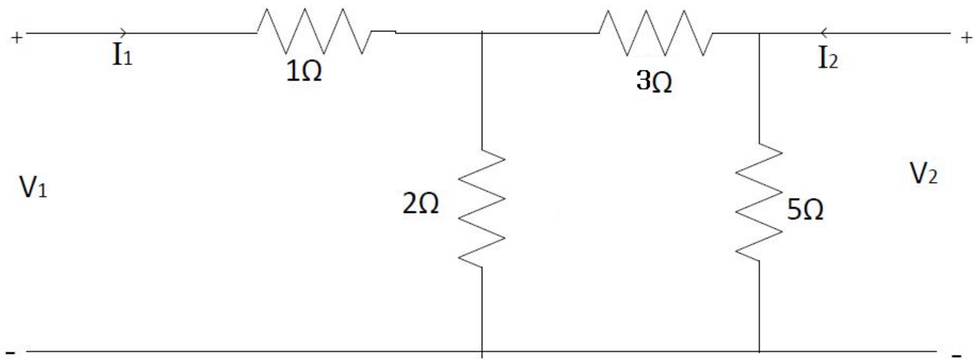


Fig 5.

Assuming first that the output is open circuited, I_2 = 0,

Looking back to the network from terminals ab,

R_{eq1} = [(3 + 5) ∥ 2] +1 = 2.6 Ω.

Let us assume I_1 to be the input current such that,

$V_1 = I_1\ R_{eq1}$ = 2.6 I_1 volt

And, $Z_{11} = \frac{V_1}{I_1}$ when I_2 = 0

$Z_{11} =$ 2.6 Ω.

Let us assume the current through 5 Ω resistor be I_x

Then, $I_x = I_1 * \frac{2}{2+3+5} = \frac{I_1}{5}$ A

And, $V_2 = \frac{I_1}{5} * 5 = I_1$ Volt

This gives, $Z_{21} = \frac{V_2}{I_1}$ when I_2 = 0

$Z_{21} =$ 1 Ω.

Next assuming the input to be open, I_1= 0. The equivalent resistance of the network looking back from the terminal gh is given by

R_{eq2} = [(3 + 2) ∥ 5] = 2.5 Ω.

Let us assume I_2 to be the input current such that,

$V_2 = I_2\ R_{eq2}$ = 2.5 I_2 volt

And, $Z_{22} = \frac{V_2}{I_2}$ when I_1 = 0

$Z_{22} =$ 2.5 Ω.

Let us assume the current through 2 Ω resistor be I_y

Then, $I_y = I_2 * \frac{5}{2+3+5} = \frac{I_2}{2}$ A

With terminal a-b open, $V_{a-b} = V_{c-d}$, i.e., the voltage across c-d terminal is V_1 too.

Hence, $V_1 = I_y * 2 = \frac{I_2}{2} * 2 = I_2$ Volt

This gives, $Z_{12} = \frac{V_1}{I_2}$ when $I_1 = 0$

$Z_{21} = 1$ Ω.

Example II: Find z parameters of the following network shown in fig 6.

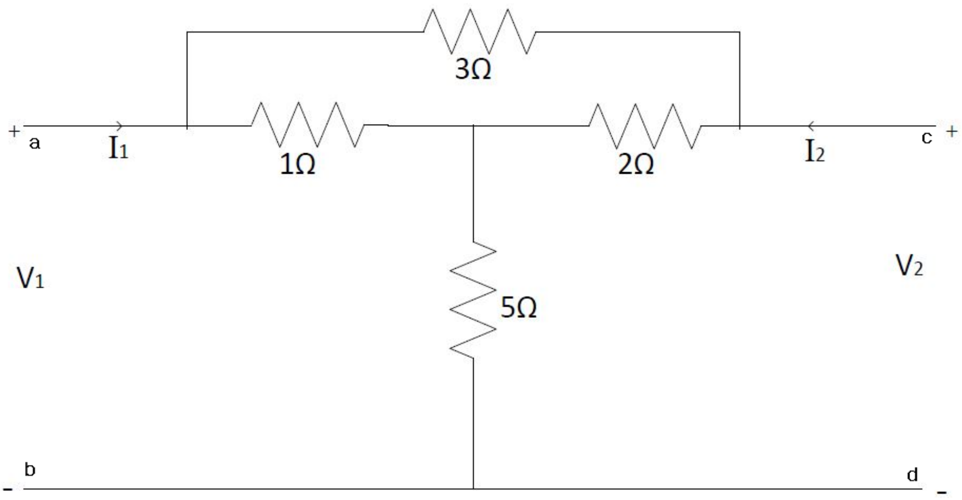


Fig 6.

Let us first assume the output terminal c-d are open circuited and V_1 is applied at a-b terminals (input port).

Looking back to the network from terminals ab,

$R_{eq1} = [(3 + 2) \parallel 1] + 5 = \frac{35}{6}$ Ω.

Let us assume I_1 to be the input current such that,

$V_1 = I_1\, R_{eq1} = \frac{35}{6}\, I_1$ volt

And, $Z_{11} = \frac{V_1}{I_1}$ when $I_2 = 0$

$Z_{11} = \frac{35}{6}$ Ω.

Then, $I_3 = I_1 * \frac{1}{2+3+1} = \frac{I_1}{6}$ A

Thus drop V_2 across terminal c-d is given by

$V_2 = I_3 * 2 + I_1 * 5 = \frac{I_1}{6} * 2 + I_1 * 5$ Volt

This gives, $Z_{21} = \frac{V_2}{I_1}$ when $I_2 = 0$

$$Z_{21} = \frac{16}{3}\ \Omega.$$

Next let us assume the oinput terminal a-b are open circuited and V_2 is applied at c-d terminals (output port).

$R_{eq2} = [(3+1) \parallel 2] + 5 = \frac{19}{3}\ \Omega.$

Let us assume I_2 to be the input current such that,

$V_2 = I_2\, R_{eq2} = \frac{19}{3}\ I_2$ volt

And, $Z_{22} = \frac{V_2}{I_2}$ when $I_1 = 0$

$$Z_{22} = \frac{19}{3}\ \Omega.$$

Then, $I_3 = I_2 * \frac{2}{2+3+1} = \frac{I_2}{3}$ A

Hence, $V_1 = \frac{I_2}{3} * 1 + 5I_2 = \frac{16}{3} I_2$ Volt

This gives, $Z_{12} = \frac{V_1}{I_2}$ when $I_1 = 0$

$$Z_{21} = \frac{16}{3}\ \Omega.$$

Y – PARAMETERS (Short Circuit Admittance Parameters):

The figure below shows a two port network. The voltages & currents of both ports are shown. The currents inward to the network are considered as positive.

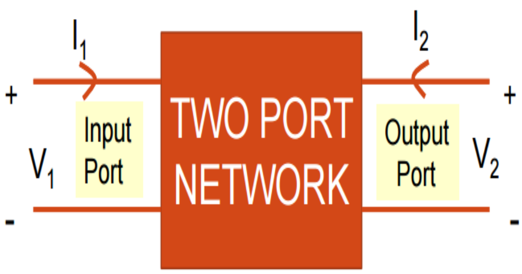


Fig 7. Two port Network

For a two port network,

[I] = [Y][V]……………………………………………………………………………………..(v)

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \text{.. (vi)}$$

Where, [Y] = $\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$ = Admittance Matrix

Equation (ii) can be written as,

$I_1 = Y_{11}V_1+Y_{12}V_2$ ....(vii)

$I_2 = Y_{21}V_1+Y_{22}V_2$ ....(viii)

To find

Y_{11} & Y_{21}, output port should be short circuited, i.e; $V_2 = 0$,

So, from equation (vii), $Y_{11} = \left[I_1/V_1 \right]_{V2 = 0}$; Known as Input driving point admittance

From equation (viii), $Y_{21} = \left[I_2/V_1 \right]_{V2 = 0}$; Known as forward transfer admittance

To find

Y_{12} & Y_{22}, input port should be short circuited, i.e; $V_1 = 0$,

So, from equation (vii), $Y_{12} = \left[I_1/V_2 \right]_{V1 = 0}$; Known as reverse transfer admittance

From equation (viii), $Z_{22} = \left[I_2/V_2 \right]_{V1 = 0}$; Known as Output driving point admittance

Y_{11}, Y_{12}, Y_{21}, Y_{22} all are admittances and unit is mho.

As Y parameters are obtained by short circuiting any of the two ports and expressed as ratio of current voltage , these are known as **SHORTCIRCUIT ADMITTANCE PARAMETERS.**

Equivalent circuit model of Y parameter representation:

The following circuit diagram shows equivalent circuit model of Y parameters which satisfies equation (vii) & (viii).

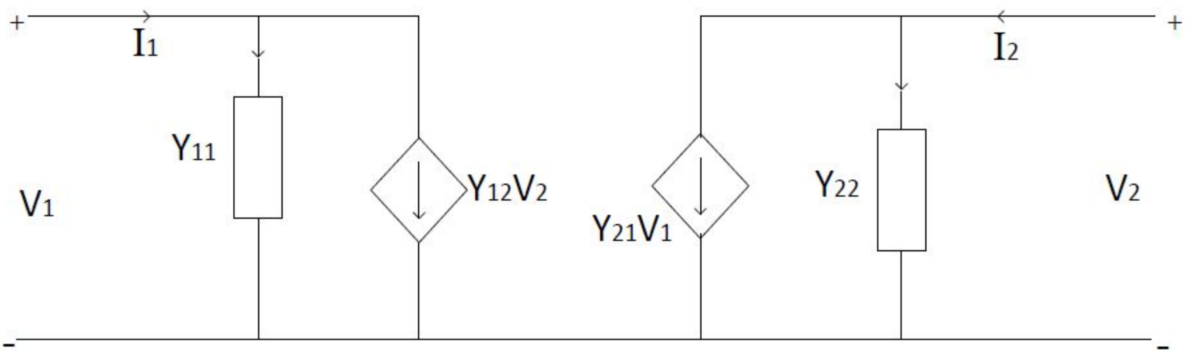


Fig 8. Equivalent circuit model of Y parameter representation

Example III: Find Y parameters of the following network shown in fig 9.

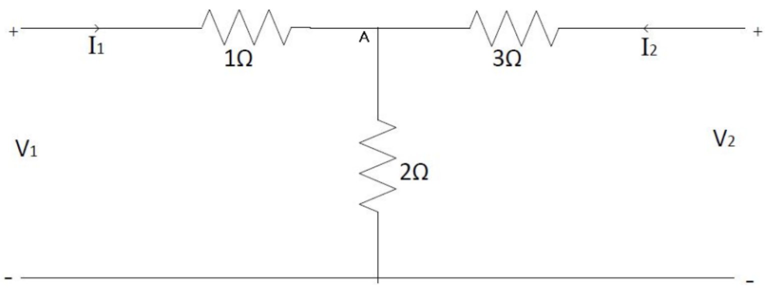


Fig 9.

Let voltage across 2 ohm resistance is V_3.

From above figure,

$I_1 = \frac{V_1 - V_3}{1}$

$I_1 = V_1 - V_3$ ……(i)

$I_2 = \frac{V_2 - V_3}{3}$ ……(ii)

Applying KCL at Node A, $I_1 + I_2 = \frac{V_3}{2}$ ……(iii)

Substituting Eqn. (i) and (ii) in Eqn. (iii),

$V_1 - V_3 + \frac{V_2 - V_3}{3} = \frac{V_3}{2}$

$V_3 = \frac{6}{11}V_1 + \frac{2}{11}V_2$ ……(iv)

Substituting Eqn. (iv) in Eqn. (i),

$I_1 = V_1 - \frac{6}{11}V_1 + \frac{2}{11}V_2$

$I_1 = \frac{5}{11}V_1 - \frac{2}{11}V_2$ ……(v)

Substituting Eqn. (iv) in Eqn. (ii),

$I_2 = \frac{V_2}{3} - \frac{1}{3}(\frac{6}{11}V_1 + \frac{2}{11}V_2)$

$I_2 = -\frac{2}{11}V_1 + \frac{3}{11}V_2$ ……(vi)

Comparing Eqn. (v) and (vi) with Y- parameter equations,

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{5}{11} & -\frac{2}{11} \\ -\frac{2}{11} & \frac{3}{11} \end{bmatrix}$$

Example IV: Obtain Y parameters of the following network shown in fig 10**.**

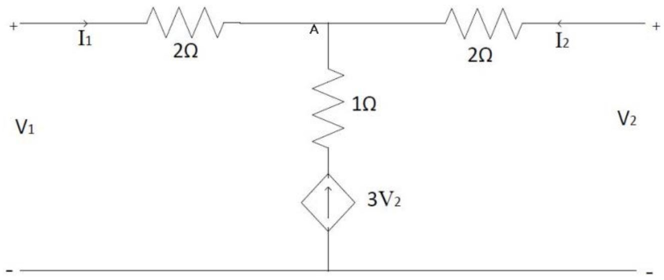


Fig 10.

Let, V_3 be voltage at node A.

From figure, we can write,

$I_1 = \frac{V_1 - V_3}{2}$

$I_1 = \frac{1}{2}V_1 - \frac{1}{2}V_3$ ……(i)

$I_2 = \frac{V_2 - V_3}{2}$ ……(ii)

Applying KCL at Node A, $I_1 + I_2 + 3V_2 = 0$ ……(iii)

Substituting Eqn. (i) and (ii) in Eqn. (iii),

$\frac{V_1 - V_3}{2} + \frac{V_2 - V_3}{2} + 3V_2 = 0$

$V_3 = \frac{1}{2}V_1 + \frac{7}{2}V_2$ ……(iv)

Substituting Eqn. (iv) in Eqn. (i),

$I_1 = \frac{1}{2}V_1 - \frac{1}{2}(\frac{1}{2}V_1 + \frac{7}{2}V_2)$

$I_1 = \frac{1}{4}V_1 - \frac{7}{4}V_2$ ……(v)

Substituting Eqn. (iv) in Eqn. (ii),

$I_2 = \frac{V_2}{2} - \frac{1}{2}(\frac{1}{2}V_1 + \frac{7}{2}V_2)$

$I_2 = -\frac{1}{4}V_1 - \frac{5}{4}V_2$ ……(vi)

Comparing Eqn. (v) and (vi) with Y- parameter equations,

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & -\frac{7}{4} \\ -\frac{1}{4} & -\frac{5}{4} \end{bmatrix}$$

h – PARAMETERS (Hybrid Parameters):

h – parameter representations are widely used in electronic circuit modelling, especially in transistor modelling. To find h parameters, both open circuit & short circuit conditions are utilized at input & output ports respectively, hence it is termed as hybrid parameters.

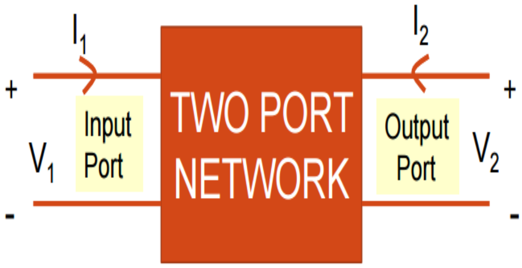


Fig 11: Two Port Network

For a two port network,

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \quad \text{(ix)}$$

Equation (ii) can be written as,

$V_1 = h_{11}I_1 + h_{12}V_2$ ....(x)

$I_2 = h_{21}I_1 + h_{22}V_2$ ....(xi)

To find

h_{11} & h_{21}, output port should be short circuited, i.e; $V_2 = 0$,

So, from equation (x), $h_{11} = \left[V_1 / I_1 \right]_{V2 = 0}$; Unit : ohm

From equation (xi), $h_{21} = \left[I_2 / I_1 \right]_{V2 = 0}$; Unit : None

To find

h_{12} & h_{22}, input port should be open circuited, i.e; $I_1 = 0$,

So, from equation (v), $h_{12} = \left[V_1 / V_2 \right]_{I1 = 0}$; Unit :None

From equation (vi), $Z_{22} = \left[I_2 / V_2 \right]_{I1 = 0}$; Unit : mho

Equivalent circuit model of h parameter representation:

The following circuit diagram shows equivalent circuit model of Y parameters which satisfies equation (x) & (xi).

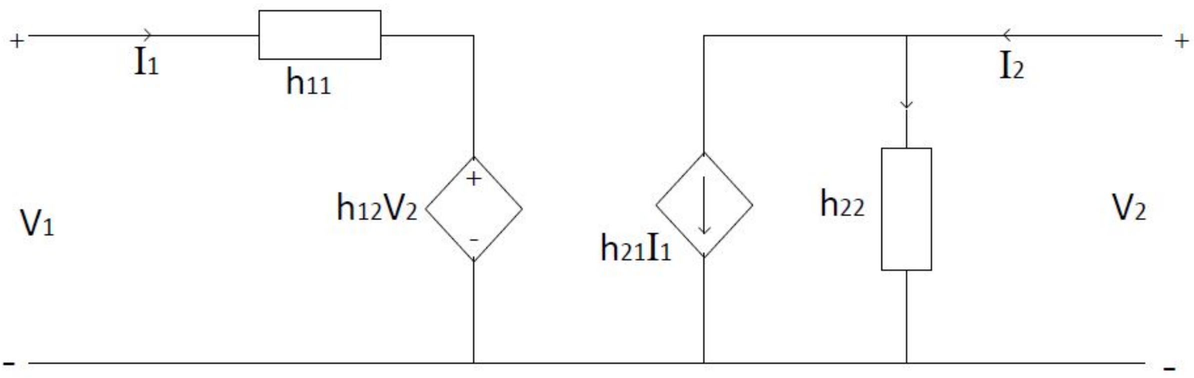


Fig 12. Equivalent circuit model of h parameter representation

ABCD PARAMETERS (Transmission line or Chin Parameters):

ABCD parameters are widely used in transmission line parameter calculations, hence these are also known as transmission line parameters.

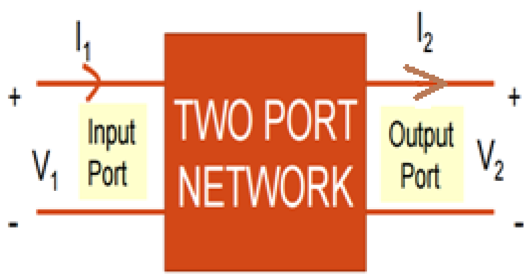


Fig 13: Two Port Network

It is observed from above figure that, I_1 is positive as it is inward to the network, whereas I_2 is negative as it is leaving the network.

For a two port network,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \quad \text{(xii)}$$

Equation (ii) can be written as,

$V_1 = AV_2 - B\,I_2$...(xiii)

$I_1 = CV_2 – D\,I_2$...(xiv)

To find

A & C, output port should be open circuited, i.e; $I_2 = 0$, So, from equation (xiii), A = $\left[{V_1}/{V_2}\right]_{I2 = 0}$; Unit : None

From equation (xiv), C = $\left[{I_1}/{V_2}\right]_{I2 = 0}$; Unit : mho

To find

B & D, output port should be short circuited, i.e; $V_2 = 0$,

So, from equation (xiii), B = $\left[{V_1}/{-I_2}\right]_{V2 = 0}$; Unit :ohm

From equation (xiv), D = $\left[{I_1}/{-I_2}\right]_{V2= 0}$; Unit : None

Example III: Find ABCD parameters of the following network shown in fig 14.

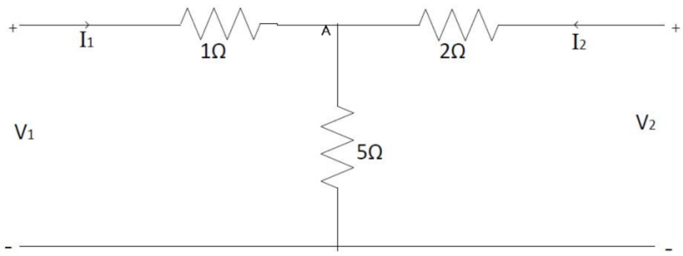


Fig 14.

Applying KVL at left mesh, $V_1 = 6I_1 + 5I_2$ ………………(i)

Applying KVL at right mesh, $V_2 = 5I_1 + 7I_2$ ………………(ii)

Hence, $5I_1 = V_2 - 7I_2$

$I_1 = \frac{1}{5}V_2 - \frac{7}{5}I_2$ ………………(iii)

Substituting Eqn. (iii) in Eqn. (i),

$V_1 = 6(\frac{1}{5}V_2 - \frac{7}{5}I_2) + 5I_2$

$= \frac{6}{5}V_2 - \frac{17}{5}I_2$ ………………(iv)

Substituting Eqn. (iv) in Eqn. (i),

$I_1 = \frac{1}{2}V_1 - \frac{1}{2}(\frac{1}{2}V_1 + \frac{7}{2}V_2)$

$I_1 = \frac{1}{4}V_1 - \frac{7}{4}V_2$ ………………(v)

Comparing Eqn. (iii) and (iv) with transmission parameter equations,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{6}{5} & \frac{17}{5} \\ \frac{1}{5} & \frac{7}{5} \end{bmatrix}$$

Condition for Reciprocity of a Two port network

A network is said to be reciprocal if the ratio of output to input remains identical even if the position of input-output are interchanged.

Condition for Reciprocity in Z parameters:

Case I:

Let, a input voltage V_1 is applied to port 1, and terminals of port 2 are short circuited. The figure 15 shows for an input voltage V_1, $-I_2$ current is flowing through port 2.

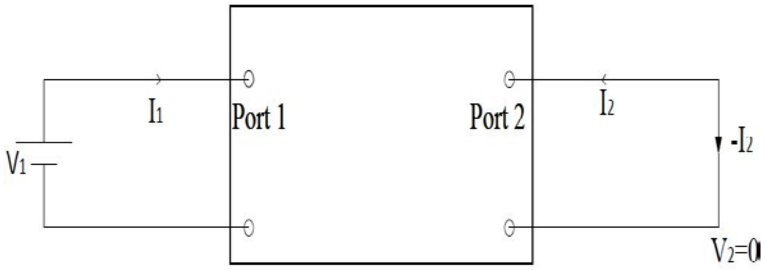


Fig 15.

From figure we can write,

$V_1 = Z_{11}I_1 - Z_{12}I_2$………………………………………………………………………….. (xv)

$V_2 = 0 = Z_{21}I_1 - Z_{22}I_2$ ……………………………………………………………………(xvi)

Solving equation (xv) & (xvi), we get

$(V_1/I_2) = \frac{Z_{21}}{\Delta Z}$………………………………………………………………………(xvii)

Where, $\Delta Z = Z_{11}Z22 - Z_{12}Z_{21}$

Case II:

When position of input& output are altered, with V_2 voltage applied as input at port 2 & port 1 is short circuited as shown in figure 16.

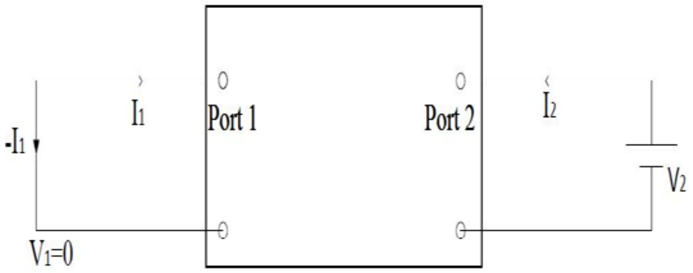


Fig 16.

From figure we can write,

$V_1 = 0 = -Z_{11}I_1 + Z_{12}I_2$……………………………………………………………….. (xviii)

$V_2 = - Z_{21}I_1 + Z_{22}I_2$ …………………………………………………………………………(xix)

Solving equation (xviii) & (xix), we get

$(V_2/I_1) = \frac{Z_{12}}{\Delta Z}$……………………………………………………………………………(xx)

Where, $\Delta Z = Z_{11}Z22 - Z_{12}Z_{21}$

For a two port network to be reciprocal, the following condition should be satisfied.

$(V_1/I_2) = (V_2/I_1)$

Hence, **$Z_{12} = Z_{21}$ …… This is condition for reciprocity in Z parameters.**

Condition for Reciprocity in Y parameters:

Case I:

Let, a input voltage V_1 is applied to port 1, and terminals of port 2 are short circuited. The figure 17 shows for an input voltage V_1, $-I_2$ current is flowing through port 2.

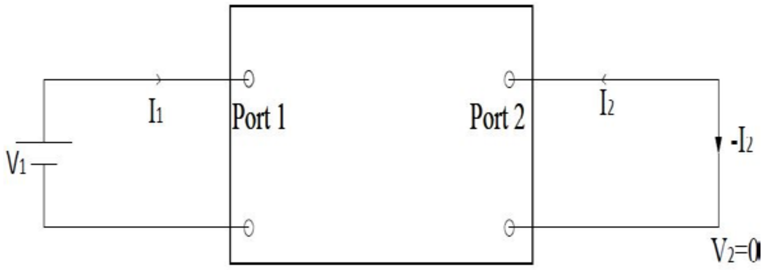


Fig 17.

From figure we can write,
$-I_2 = Y_{21}V_1$ ……………………………………………………………………………………(xxi)

Or, $(V_1/ I_2) = -Y_{21}$ ……………………………………………………………………………….(xxii)

Case II:

When position of input& output are altered, with V_2 voltage applied as input at port 2 & port 1 is short circuited as shown in figure 18.

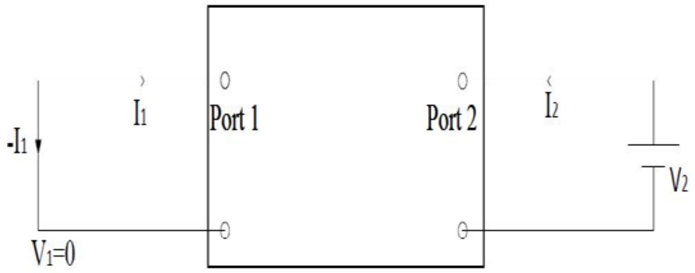


Fig 18.

From figure we can write,
$-I_1 = Y_{12}V_2$…………………………………………………………………………………... (xxiii)

Or, $(V_2/ I_1) = -Y_{12}$ …………………………………………………………………………....(xxiv)

For a two port network to be reciprocal, the following condition should be satisfied.

$(V_1/I_2) = (V_2/I_1)$

Hence, **$Y_{12} = Y_{21}$ …… This is condition for reciprocity in Y parameters.**

Condition for Reciprocity in h parameters:

Case I:

Let, a input voltage V_1 is applied to port 1, and terminals of port 2 are short circuited. The figure 19 shows for an input voltage V_1, $-I_2$ current is flowing through port 2.

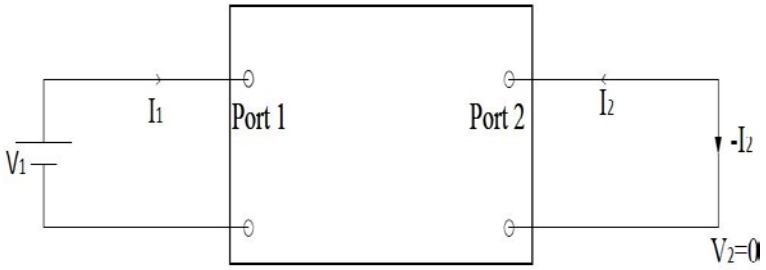


Fig 19.

From figure we can write,

$V_1 = h_{11}I_1$ …………………………………………………………………………………….. (xxv)

$-I_2 = h_{21}I_1$ ……………………………………………………………………………………..(xxvi)

Solving equation (xxv) & (xxvi), we get

$(V_1/I_2) = -\frac{h_{11}}{h_{21}}$………………………………………………………………………………(xxvii)

Case II:

When position of input& output are altered, with V_2 voltage applied as input at port 2 & port 1 is short circuited as shown in figure 20.

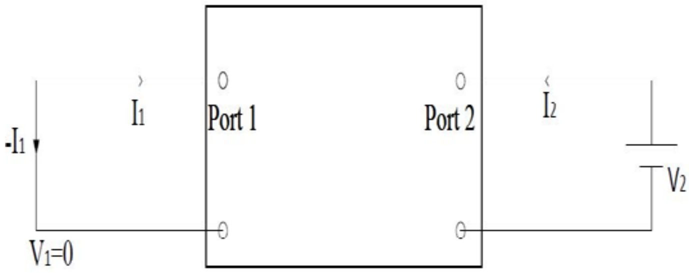


Fig 20.

From figure we can write,

$V_1 = 0 = -h_{11}I_1 + h_{12}V_2$…………………………………………………………………………….. (xxviii)

$I_2 = -h_{21}I_1 + h_{22}V_2$ …………………………………………………………………………(xxix)

Solving equation (xxviii) & (xxix), we get

$(V_2/I_1) = \frac{h_{11}}{h_{12}}$…………………………………………………………………………………(xxx)

For a two port network to be reciprocal, the following condition should be satisfied.

$(V_1/I_2) = (V_2/I_1)$

Hence, $\mathbf{h_{12} = -h_{21}}$ **…… This is condition for reciprocity in h parameters.**

Condition for Reciprocity in ABCD parameters:

Case I:

Let, a input voltage V_1 is applied to port 1, and terminals of port 2 are short circuited. The figure 21 shows for an input voltage V_1, $-I_2$ current is flowing through port 2.

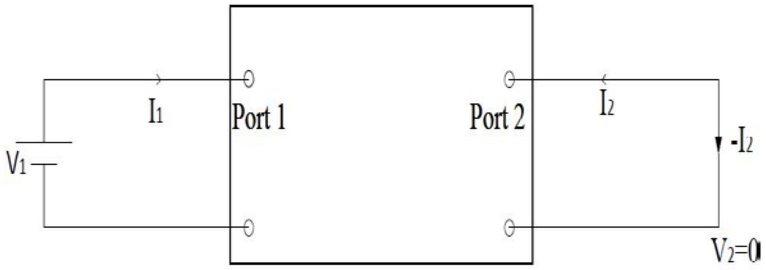


Fig 21.

From figure we can write,

$V_1 = -BI_2$ ………………………………………………………………………………….. (xxxi)

$(V_1/I_2) = -B$ ……………………………………………………………………………………(xxxii)

Case II:

When position of input& output are altered, with V_2 voltage applied as input at port 2 & port 1 is short circuited as shown in figure 22.

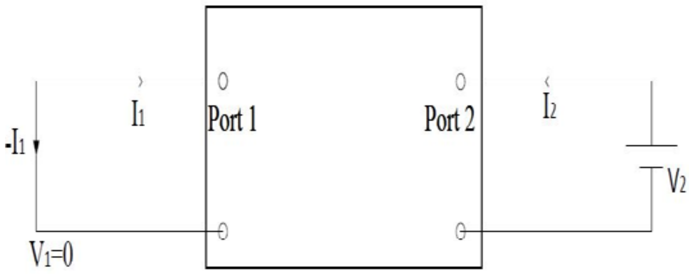


Fig 22.

From figure we can write,

$V_1 = 0 = AV_2 - B\,I_2$ ……………………………………………………………………………….. (xxxiii)

$I_1 = CV_2 - DI_2$ ……………………………………………………………………………………(xxxiv)

Solving equation (xxxiii) & (xxxiv), we get

$(V_2/I_1) = -\frac{B}{AD-BC}$………….……………………………………………………………………(xxxv)

For a two port network to be reciprocal, the following condition should be satisfied.

$(V_1/I_2) = (V_2/I_1)$

Hence, **AD - BC= 1 …… This is condition for reciprocity in ABCD parameters.**

Condition for symmetry in Two Port Network

A two port network is said to be symmetrical if input & output port can be interchanged without altering port voltages & currents.

Condition for symmetry in Z parameters:

Case I:

Let, V voltage is applied to input port P1 & output port P2 is kept open circuited as shown in fig. 23;

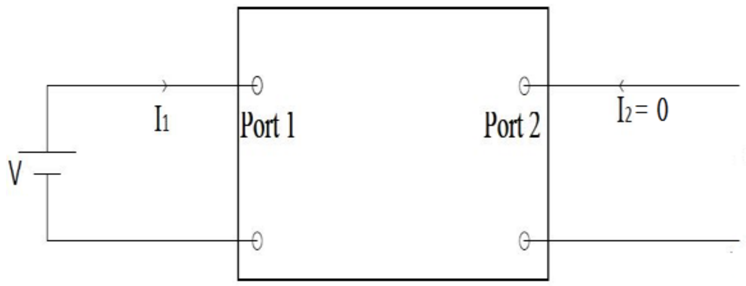


Fig 23.

From above figure, we can write,

$V = Z_{11}I_1$ ……………………………………………………………………………………….(xxxvi)

Case II:

When the input & output ports are interchanged, i.e; if V voltage is now applied at port 2 & port 1 is kept open circuited as shown in fig 24.

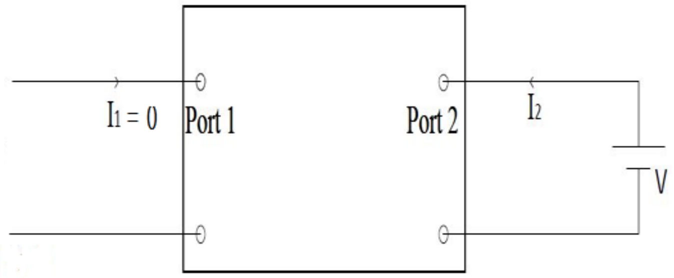


Fig 24.

Then, we can write,

$\mathbf{V = Z_{22}I_2}$ ……………………………………………………………………………………….(xxxvii)

For a two port network to be symmetrical, for a particular voltage (V) applied to port 1 & port 2 respectively

$I_1 = I_2$

From equation (xxxvi) & (xxxvii), we can write, for symmetry

$\mathbf{Z_{11} = Z_{22}}$……… This is condition for symmetry in Z parameters.

Condition for symmetry in Y parameters:

Case I:

Let, V voltage is applied to input port P1 & output port P2 is kept open circuited as shown in fig. 25;

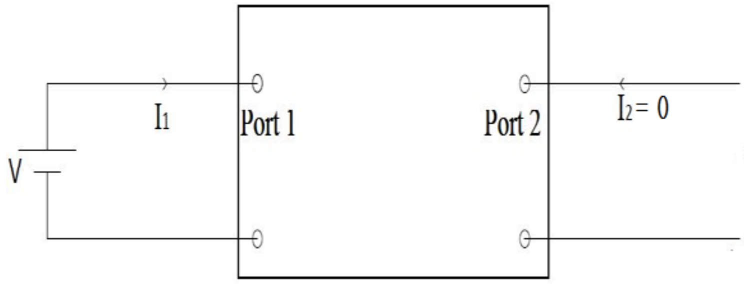


Fig 25.

From above figure, we can write,

$I_1 = Y_{11}V$ ………………………………………………………………………………………….(xxxviii)

Case II:

When the input & output ports are interchanged, i.e; if V voltage is now applied at port 2 & port 1 is kept open circuited as shown in fig 26.

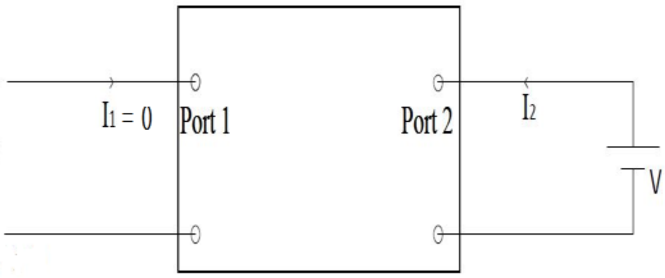


Fig 26.

Then, we can write,

$I_2 = Y_{22}V$………………………………………………………………………………………….(xxxix)

For a two port network to be symmetrical, for a particular voltage (V) applied to port 1 & port 2 respectively

$I_1 = I_2$

From equation (xxxviii) & (xxxix), we can write, for symmetry

$\mathbf{Y_{11} = Y_{22}}$……… This is condition for symmetry in Y parameters.

Condition for symmetry in ABCD parameters:

We know, for symmetry

$Z_{11} = Z_{22}$;

$Z_{11} = \left[\frac{V_1}{I_1}\right]_{I2=0} = \left[\frac{AV_2 - BI_2}{CV_2 - DI_2}\right]_{I2=0} = \left[\frac{A}{C}\right]$…………………………………………………………… (xxxix)

We know,

$Z_{22} = \left[\frac{V_2}{I_2}\right]_{I1 = 0}$

Hence, with $I_1 = 0$;

$I_1 = 0 = CV_2 – DI_2$

$\left[\frac{V_2}{I_2}\right] = \left[\frac{D}{C}\right]$

Or, $Z_{22} = \left[\frac{V_2}{I_2}\right]_{I1 = 0} = \left[\frac{D}{C}\right]$………………………………………………………………………….. (xxxx)

Comparing equation (xxxix) & (xxxx)

We can write, for symmetrical network,

$Z_{11} = Z_{22}$

Or, $\left[\frac{A}{C}\right] = \left[\frac{D}{C}\right]$

Or, A = D ……………….. This is condition for symmetry in ABCD parameters.

Interrelation between two port network parameters

Z- Parameters in Terms of Y Parameters

We know, $[Z] = [Y]^{-1}$

Or, $\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}^{-1}$

By simplifying, we get,

$Z_{11} = \frac{Y_{22}}{\Delta Y}$; $Z_{12} = - \frac{Y_{12}}{\Delta Y}$; $Z_{21} = - \frac{Y_{21}}{\Delta Y}$; $Z_{11} = \frac{Y_{11}}{\Delta Y}$

Here, $\Delta Y = Y_{11} Y_{22} - Y_{12} Y_{21}$

Z- Parameters in Terms of ABCD Parameters

From the governing equation of ABCD parameters,

$V_1 = AV_2 –BI_2$…………………………………………………………………………….(xxxxi)

$I_1 = CV_2 – DI_2$ ……………………………………………………………………….(xxxxii)

From equation (xxxxii),

$V_2 = \frac{1}{C} . I_1 + \frac{D}{C} . I_2$……………………………………………………………………….(xxxxiii)

Similarly, from From equation (xxxxi),

$V_1 = [\frac{1}{C} . I_1 + \frac{D}{C} . I_2] A – BI_2$

$= \frac{A}{C} . I_1 + \frac{AD-BC}{C} . I_2$……………………………………………………………..…….(xxxxiv)

Comparing equations (xxxxiv) & (xxxxiii) with the governing equations of Z- parameter network,

We can write

$Z_{11} = \frac{A}{C}$, $Z_{12} = \frac{AD-BC}{C}$

$Z_{21} = \frac{1}{C}$, $Z_{22} = \frac{D}{C}$.

Z-Parameters in Terms of Hybrid Parameters

The governing equations of the h-parameter network are given by

$V_1 = h_{11} I_1 + h_{12}V_2$……………………………………………………………..…….(xxxxv)

$I_2 = h_{21}I_1 + h_{22}V_2$……………………………………………………………………….(xxxxvi)

From equation (xxxxvi),

$V_2 = [- \frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} . I_2]$ ……………………………………………………………….….(xxxxvii)

$V_1 = h_{11}I_1 + h_{12}V_2 = h_{11}I_1 + h_{12} [- \frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} . I_2]$

$V_1 = \frac{\Delta h}{h_{22}} I_1 + \frac{h_{12}}{h_{22}} . I_2$…………………………………………………………...….(xxxxviii)

Comparing equations (xxxxvii) & (xxxxviii) with the governing equations of Z- parameter network,

We can write

$Z_{11} = \frac{\Delta h}{h_{22}}$, $Z_{12} = \frac{h_{12}}{h_{22}}$

$Z_{21} = - \frac{h_{21}}{h_{22}}$, $Z_{22} = \frac{1}{h_{22}}$.

Here, $\Delta h = h_{11} h_{22} - h_{12} h_{21}$

Similarly, interrelationship for other network parameters can be established.

Y parameters in terms of other network parameters:

Y parameters	In terms of Z parameters	In terms of ABCD parameters	In terms of h parameters
Y_{11}	$\frac{Z_{22}}{\Delta Z}$	$\frac{D}{B}$	$\frac{1}{h_{11}}$
Y_{12}	$- \frac{Z_{12}}{\Delta Z}$	$- \frac{AD-BC}{B}$	$- \frac{h_{12}}{h_{11}}$
Y_{21}	$- \frac{Z_{21}}{\Delta Z}$	$- \frac{1}{B}$	$\frac{h_{21}}{h_{11}}$
Y_{22}	$\frac{Z_{11}}{\Delta Z}$	$\frac{A}{B}$	$\frac{\Delta h}{h_{11}}$

ABCD parameters in terms of other network parameters:

ABCD parameters	In terms of Z parameters	In terms of Y parameters	In terms of h parameters
A	$\frac{Z_{11}}{Z_{21}}$	$-\frac{Y_{22}}{Y_{21}}$	$-\frac{\Delta h}{h_{21}}$
B	$\frac{\Delta Z}{Z_{21}}$	$-\frac{1}{Y_{21}}$	$-\frac{h_{11}}{h_{21}}$
C	$\frac{1}{Z_{21}}$	$-\frac{\Delta Y}{Y_{21}}$	$-\frac{h_{22}}{h_{21}}$
D	$\frac{Z_{22}}{Z_{11}}$	$-\frac{Y_{11}}{Y_{21}}$	$-\frac{1}{h_{21}}$

h parameters in terms of other network parameters:

h parameters	In terms of Z parameters	In terms of Yparameters	In terms of ABCD parameters
h_{11}	$\frac{\Delta Z}{Z_{22}}$	$\frac{1}{Y_{11}}$	$\frac{B}{D}$
h_{12}	$\frac{Z_{12}}{Z_{22}}$	$-\frac{Y_{12}}{Y_{11}}$	$\frac{AD - BC}{D}$
h_{21}	$-\frac{Z_{21}}{Z_{22}}$	$\frac{Y_{21}}{Y_{11}}$	$-\frac{1}{D}$
h_{22}	$\frac{1}{Z_{22}}$	$\frac{\Delta Y}{Y_{11}}$	$\frac{C}{D}$

Interconnection of two port networks

Two-port networks may be interconnected in various configurations, such as series, parallel, series-parallel, and parallel-series connections. For each configuration a certain set of parameters may be more useful than others to describe the network.

Figure below shows a series connection of two-port networks N_a and N_b.

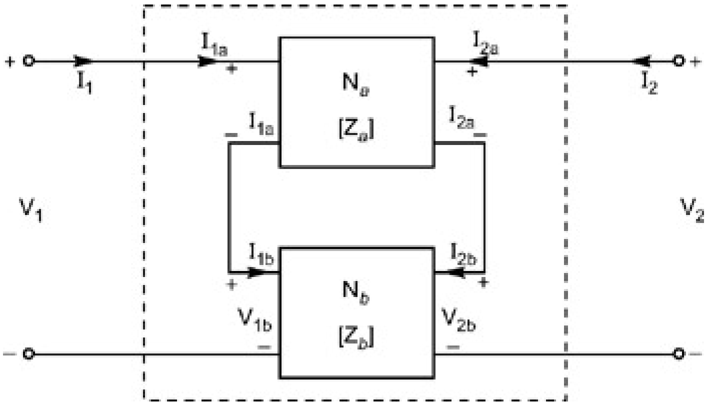


Figure 27: Series connection of two two-port networks

For network N_a,

$$\begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix} = \begin{bmatrix} Z_{11a} & Z_{12a} \\ Z_{21a} & Z_{22a} \end{bmatrix} \begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix}$$

$$V_{1a} = Z_{11a}I_{1a} + Z_{12a}I_{2a}$$

$$V_{2a} = Z_{21a}I_{1a} + Z_{22a}I_{2a}.$$

For network N_b,

$$\begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix} = \begin{bmatrix} Z_{11b} & Z_{12b} \\ Z_{21b} & Z_{22}b \end{bmatrix} \begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix}$$

$$V_{1b} = Z_{11b}I_{1b} + Z_{12b}I_{2b}$$

$$V_{2b} = Z_{21b}I_{1b} + Z_{22b}I_{2b}$$

The condition for series connection is

$$I_{1a} = I_{1b} = I_1, \ \textit{and} \ I_2 = I_{2b} = I_2 \ (\textit{current same})$$

$$V_1 = V_{1a} + V_{1b}$$

$$V_2 = V_{2a} + V_{2b}$$

Putting the values of V_{1a} and V_{1b} from Equation (10.62) and Equation (10.64),

$$\begin{aligned} V_1 &= Z_{11a}I_{1a} + Z_{12a}I_{2a} + Z_{11b}I_{1b} + Z_{12b}I_{2b} \\ &= Z_{11a}I_1 + Z_{12a}I_2 + Z_{11b}I_1 + Z_{12b}I_2 \ [I_{1a} = I_{1b} = I_1, \ I_{2a} = I_{2b} = I_2] \\ V_1 &= (Z_{11a} + Z_{11b})I_1 + (Z_{12a} + Z_{12b})I_2. \end{aligned}$$

Putting the values of V_{2a} and V_{2b} from Equation (10.63) and Equation (10.65) into Equation (10.67), we get

$$V_2 = (Z_{21a} + Z_{21b})I_1 + (Z_{22a} + Z_{22b})I_2$$

The Z-parameters of the series-connected combined network can be written as

$$\begin{aligned} V_1 &= Z_{11}I_1 + Z_{12}I_2 \\ V_2 &= Z_{21}I_1 + Z_{22}I_2, \end{aligned}$$

where

$$Z_11 = Z_{11a} + Z_{11b}$$
$$Z_12 = Z_{12a} + Z_{12b}$$
$$Z_21 = Z_{21a} + Z_{21b}$$
$$Z_22 = Z_{22a} + Z_{22b}$$

or in the matrix form,

$$[Z] = [Z_a] + [Z_b].$$

The overall Z-parameter matrix for series connected two-port networks is simply the sum of Z-parameter matrices of each individual two-port network connected in series.

Parallel Connection

Figure below shows a parallel connection of two two-port networks N_a and N_b. The resultant of two admittances connected in parallel is $Y_1 + Y_2$. So in parallel connection, the parameters are Y-parameters.

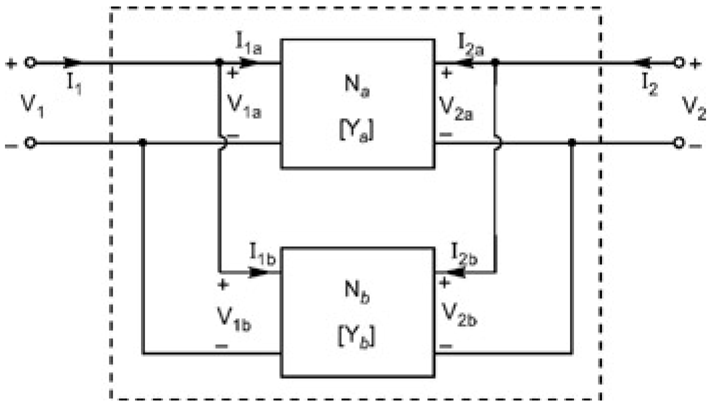


Figure 28: Parallel connections for two two-port networks

For network N_a,

$$\begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix} = \begin{bmatrix} Y_{11a} & Y_{12a} \\ Y_{21a} & Y_{22a} \end{bmatrix} \begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix}$$

or

$$I_{1a} = Y_{11a}V_{1a} + Y_{12a}V_{2a}$$

$$I_{2a} = Y_{21a}V_{1a} + Y_{22a}V_{2a}.$$

For network N_b,

$$\begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix} = \begin{bmatrix} Y_{11b} & Y_{12b} \\ Y_{22b} & Y_{22b} \end{bmatrix} \begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix}$$

$$I_{1b} = Y_{11b}V_{1b} + Y_{12b}V_{2b}$$

$$I_{2b} = Y_{21b}V_{1b} + Y_{22b}V_{2b}$$

Now the condition for parallel,

$$V_{1a} = V_{1b} = V_1, \; V_{2a} = V_{2b} = V_2 \; [\textit{Same voltage}]$$

and

$$I_1 = I_{1a} + I_{1b}$$

$$I_2 = I_{2a} + I_{2b}$$

$$\begin{aligned} I_1 &= Y_{11a}V_{1a} + Y_{12a}V_{2a} + Y_{11b}V_{1b} + Y_{12b}V_{2b} \\ &= Y_{11a}V_1 + Y_{12a}V_2 + Y_{11b}V_1 + Y_{12b}V_2 \end{aligned}$$

$$I_1 = (Y_{11a} + Y_{11b})V_1 + (Y_{12a} + Y_{12b})V_2$$

Similarly,

$$I_2 = (Y_{21a} + Y_{21b})V_1 + (Y_{22a} + Y_{22b})V_2$$

The Y-parameters of the parallel connected combined network can be written as

$$\begin{aligned} I_1 &= Y_{11}V_1 + Y_{12}V_2 \\ I_2 &= Y_{21}V_1 + Y_{22}V_2 \end{aligned}$$

Comparing the above equation with...

$Y_{11} = Y_{11a} + Y_{11b}$

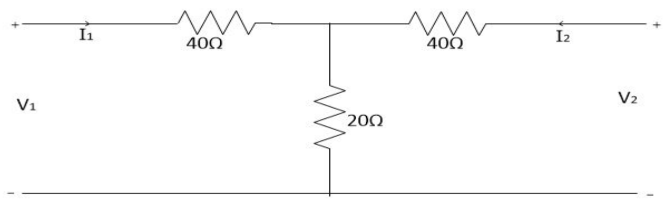
$Y_{12} = Y_{12a} + Y_{12b}$

$Y_{21} = Y_{21a} + Y_{21b}$

$Y_{22} = Y_{22a} + Y_{22b}$

Exercise:

1. Which among the following represents the precise condition of reciprocity for transmission parameters?
   A) AB - CD = 1, B) AD – BC = 1, C) AC – BD = 1, D) None of these.
2. Which is the correct condition of symmetry observed in z-parameters?
   A) $z_{11} = z_{22}$, B) $z_{11} = z_{12}$, C) $z_{12} = z_{22}$, D) $z_{12} = z_{21}$
3. Which elements act as an independent variables in Y-parameters?
   A) Current, B) Voltage, C) Both A and B, D) None of these.
4. What are transmission parameters? Where are they most effectively used? Establish, for two-port networks, the relationship between the transmission parameters and the open circuit impedance parameters.
5. Prove that for a symmetrical two-port network, $\Delta h = (h_{11}h_{22} - h_{12}h_{21}) = 1$.
6. Define z-parameters and y-parameters of a typical four terminal network. Determine the relationship between the z-parameters and y-parameters.
7. In a two port network, $Z_{11}=50\Omega$, $Z_{21}=60\Omega$, $Z_{12}=60\Omega$, $Z_{22}=25\Omega$. Compute Y parameters.
8. For the symmetrical two port network, calculate the z-parameters and ABCD parameters.



9. Test results for two-port network are
   (a) port 2 open circuited, $I_1=0.02\angle 0°A, V_1=1.4\angle 45°V, V_2=2.3\angle -26.4°V$
   (b) port 1 open circuited, $I_2=0.01\angle 0°A, V_1=1\angle -90°V, V_2=1.5\angle -53.1°V$

The source frequency in both the tests was 1000Hz. Find z-parameters.